KÁRMÁN AND THE DEVELOPMENT OF THE MATERIAL MODELS OF CONCRETE

Andor Windisch

The failure models developed until the 1960ies were defined by the testing equipment: the triaxial loading cell which was developed at the beginning of the last century by Kármán. The axial loading was performed with a solid loading plate, the central-symmetric transverse loading through hydraulic pressure. Therefore, the characterization of the failure surface with the hydrostatic normal stress and octahedral shear stress without any reference to deformations was a logical consequence.

In 1963 Hilsdorf proposed a brush-type loading equipment. Using brushes Kupfer carried out his well-known biaxial loading tests which made possible the characterization of concrete strength by means of the principal stresses. In 1977 Ottosen applied in his model for multiaxial strength of concrete the stress invariants. The same did CEB in the Bulletin d'Information N°. 156.

Van Mier (1984) applied brushes and proposed a 3D-type representation using contour lines.

The MC2010 returned to the Ottosen model and declared concrete as frictional material.

Using the principal stresses, a new, transparent (and physically really sound) form of representation of the failure surface showing the strength increase due to bi- and triaxial loading is presented.

1. RESEARCH SIGNIFICANCE

Kármán's achievements in the aerodynamics are well known but in materials science (e.g. theory of elasticity and plasticity) are less, maybe because these early findings were published in Hungarian and German. Besides developing the triaxial loading equipment in 1910 he pointed out the shortages of Mohr's theory. Although, his triaxial equipment could not produce an independent intermediate principal stress, Kármán emphasized the importance of this stress at the development of concrete strength. The renunciation from the principal stresses to the hydrostatic/octahedral stresses and to the intransparent and material-strange (but elegant) stress invariants governed until MC2010 the characterization of concrete behavior. It is high-time to return in MC2020 to the real material characteristics: concrete „knows" the principal stresses only. This allows a transparent computer/material-friendly description of concrete, high- and ultrahigh strength concrete, too. The paper presents a new type of description of 2D and 3D strength of concrete.

2. KÁRMÁN'S THEORY OF MATERIALS AND HIS TRIAXIAL LOADING EQUIPMENT

Before reporting on the development of his triaxial loading equipment and the results of his related experiments, Kármán (1910) overviews the relevant design hypotheses:

„The question to which I would like to contribute is what is the allowable level of loading? If we know the maximum allowable value of stress or elongation for a material in the case of simple tensile or compressive loading, how can we deduce from this a different kind of stress, e.g. in case of torsion or a complex stress state? The permissible stress for brittle materials is usually based on the fracture, while for plastic and ductile materials it is based on the limit of elasticity, therefore judging the allowable stress requires the knowledge of the limit of elasticity and of the fracture. Since all stress states can be characterized by the values of the three principal stresses, the question is, which function of these three is governing these limits?"

He identified that neither the highest principal stress, nor the deformation work as hypothesis are suitable for determination of the acceptable degree of exploitation.

„In the following, I would like to briefly describe the current hypotheses, highlighting the points that have not yet been decided, which have provided an opportunity for my later experiments:
- the highest principal stress guides the limit of elasticity (Lamé, Clapeyron)
- deformation work is the right measure of degree of loading.

Both hypotheses are contradicted by the simple fact that in a hydrostatic load condition, the material can be subjected to much greater stress and a much larger amount of work can be accumulated in it without reaching the limit of elasticity as one-way compression.

The remaining two hypotheses are

a) largest linear dimension change (which origins from de Saint Venant). Even if at that time different limit values were proposed for the elongation and shortening, the test results contradict this theory.
b) Coulomb's theory (improved by Mohr): the elastic limit depends on the two extreme principal stresses only. The mathematical form of the limit surface is

$$|\tau| = s_0 \pm f \sigma \qquad (1)$$

where
τ, σ the shear and normal stresses, resp.
s_0 the constant term of frictional resistance
f friction coefficient."

Kármán asks the question:
„To determine whether the stress state determined by the three principal stresses falls within the elastic limit, it is necessary to examine whether Equ. (1) exists for all directions of displacements? After a simple transformation, we get the expression where it can be seen that the intermediate principal stress is meaningless. Experiments with ductile and plastic materials (iron, copper) largely confirmed the Mohr-Coulomb theory. The mean values of the results of Bauschinger's tensile, compressive and shear experiments with cementitious specimens were to some extent consistent with the demands of Mohr's theory. However, the individual values themselves have such a great scatter, that these experiments hardly can be regarded as demonstrative."

Kármán payed special attention to the tests of Föppl, who examined whether the value of the intermediate principal stress was really irrelevant to the compressive strength. For this purpose, in addition to the ordinary compression test, where only one principal stress is not zero, the materials are subjected to a stress state having equal pressures in two perpendicular directions and zero in the third direction only. Föppl's experiments have shown that the two strengths are nearly equal when a lubricant is applied to reduce the friction between the pressure plates and the specimen, but are very different when the specimen is in direct contact with the pressure plates. Kármán did not regard these results of Föppl as entirely convincing, especially since his experiments in this regard led to the opposite result.

Kármán envisaged to answer the following questions:
a) In which cases does the limit of elasticity depend only on the maximum and minimum principal stresses, in which cases has the intermediate principal stress influence, or in which cases is the difference of the two extreme principal stresses governing?
b) In which cases does a tensile failure and in which cases does a slippage occur, i.e. what conditions should exist between the principal stresses, that one or the other case occur?

The following objectives governed the design of the experimental equipment: the principal stresses could be changed independently, preferably with a homogeneous stress distribution. Instead of combining various loading types (e.g., axial tension, twisting, internal pressure, as other researcher did), experiments with compressive and tensile axial loading were designed with simultaneous application of a uniform transverse fluid pressure. With this arrangement, it was possible to create two rows of states stress: either the two smaller or the two higher principal stresses were equal
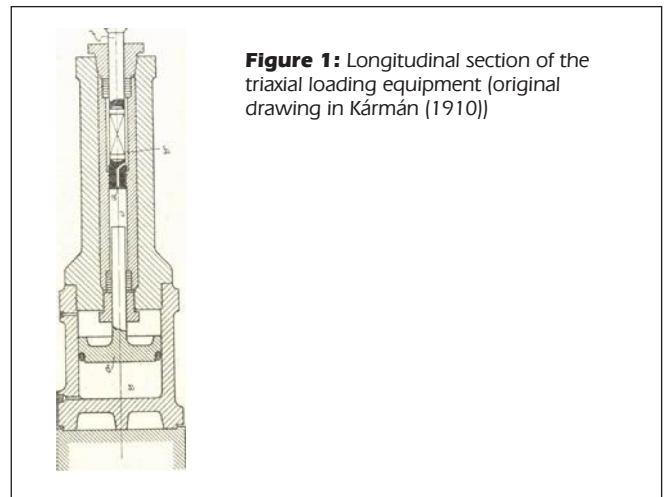
Figure 1: *Longitudinal section of the triaxial loading equipment (original drawing in Kármán (1910))*

(considering the compression stress positive) the former experiments were called compression tests, the latter tensile tests.

Although, the experiments carried out with the two equal principal stresses are not able to determine the final questions, nevertheless Kármán hoped to receive (at least) guidance on general tests with unequal principal stresses.

The equipment was produced by the steel company Krupp, based on Kármán's designs. *Fig. 1* shows the longitudinal section of the device. The diameter of the hole in the inner tube was 50 mm and the upper limit of the transverse pressure was 6000 atm. Pressure was measured with a manometer, longitudinal deformation indirectly with two micrometer screws. The diameter of the marble and sandstone specimens was 40 mm, their length was between 100 and 110 mm.

During the tests no direct observations of the specimens were possible.

A brief summary of the results of the compression tests: in these cases neither the difference

$$\sigma_1 - \sigma_2, \quad \text{nor} \quad \sigma_1 - \lambda\, \sigma_2 = \text{const.}$$

characterize the limit of elasticity.

In his second paper dated 1915, Kármán adds the question: which states of stresses cause the failure of the material?

„If we interpret the three principal stresses as spatial coordinates, a certain state of stress corresponds to every point in the space. Thus, in this representation, the elastic limit corresponds to a surface which encompasses the portion of space representing the stress states associated with the purely elastic deformation. This surface is commonly referred to as the limit surface of elasticity. Similarly, a surface is given by the stress states in which the continuity of the material ceases. This second interface can be called the fracture interface.

His conclusions are:
• The elastic limit or the compressive strength varies with the lateral pressure. The elastic limit at low values of the difference of the principal stresses is approximately proportional to the transverse pressure, later this influence decreases, at high lateral pressure it depends only on the difference of the principal stresses.
• The deformation curve of the material depends on the magnitude of the transverse pressure.
• The characteristic angles of the surface drawings are not characteristics of the material, but of the stress state at limit state of elasticity.
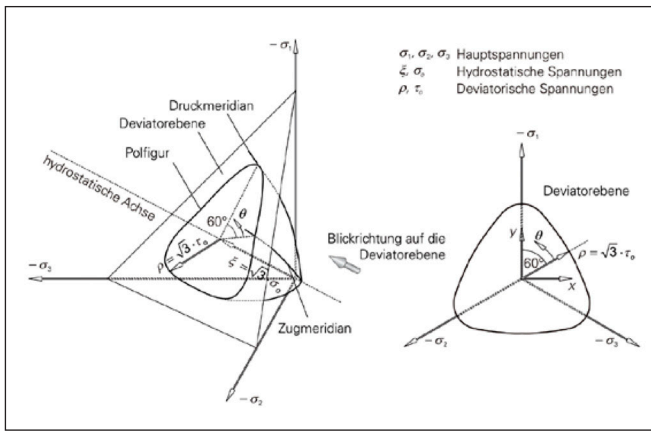
Figure 2: Designations in 3D stress space (Hauptspannungen: principal stresses; Deviatorebene: deviator plane; Blickrichtung auf die Deviatorebene: outlook to deviator plane)

- The formation of surface drawings is related to the inhomogeneity of the material and the nature of the deformation curve and can be explained by the distribution of the residual deformation.
- The moment of fracture is not independent of the flexibility of the testing machine and the clamping of the specimen, and the speed of the loading (and unloading)."

It should be noted here, that the influence of the friction between the load piston surface and the specimen are indisputable.

Concerning the further parts of this paper, it should be diagnosed, that in 1910 Kármán already knew that the Mohr-Coulomb-theory does not fit for concrete-like materials and the stress state must be characterized through the three principal stresses.

3. HISTORICAL REVIEW

The failure models developed until the '60ies of the last century were defined by the testing equipment: the triaxial loading cell, developed by Kármán. As the transverse stresses (two of the three principal stresses) were equal, hence the threefold discrete rotational symmetrical characterization of the failure surface with the hydrostatic normal stress and octahedral shear stress was a logical (but extremely intransparent) consequence. *Fig. 2* shows the basic notions and designations of this type of characterization. The description of the results of the tests with a Kármán-type equipment with the compressive and tensile meridians was also obvious:

Compressive meridian: $\sigma_1 = \sigma_2 > \sigma_3$ with $\sigma_1 \geq \sigma_2 \geq \sigma_3$
Tensile meridian: $\sigma_1 > \sigma_2 = \sigma_3$ with $\sigma_1 \geq \sigma_2 \geq \sigma_3$

Fig. 3 shows the relative positions of the measured values and the mean values vs. a 3D failure criterion: these do not fit at all. *Fig. 3* reveals how intransparent this type of representation is! Try to determine the 2D- or 3D strength for given $\sigma_{1u} / \sigma_{3u}$ and $\sigma_{2u} / \sigma_{3u}$ ratios!

4. CEB BULLETIN NO. 156

In the CEB Bulletin No. 156 the description of "the Ultimate Strength Surface (USS) is based on the following technical considerations and rational reasoning:
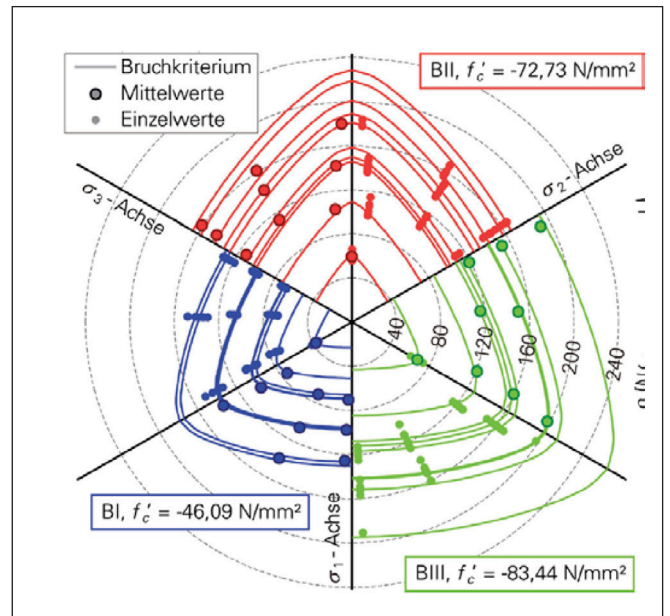
Figure 3: Relative positions of the measured and the mean values vs. a 3D octahedral failure criterion for three concretes as measured by Speck

a. USS is to be described by invariants of the stress tensors or by expressions derived from it
b. For an isotropic material without any history, the USS in the deviatoric plane (polar figure) is three-fold symmetric with respect to the hydrostatic axis.
c. Theory of plasticity and more recent fracture mechanics studies require the polar figures to be convex.
d. For a material whose uniaxial compressive strength differs from its tensile strength, one must distinguish between a triaxial compression curve and a triaxial tension curve."

Comments of the author:

To a): as well known, the invariants of a tensor (here the stress tensor) do not change with the rotation of the coordinate system. The proposed description of USS by invariants makes the stress state non-transparent and less practical. Much better is to transfer any stress state into its principal stresses with the corresponding directions. The material concrete "perceives" principal stresses only. The description by hydrostatic normal stress and octagonal shear stress was essential as at the early experiments in the triaxial cells only one principal stress was explicit; all transverse directions were "principal" directions. The hydrostatic and octagonal stress components "helped" to overcome this "difficulty".

To b): due to its production technology (pouring) concrete is not isotropic. In the era of the high capacity computers there is no reason to adhere to the not existing material isotropy.

To c): recent test series (van Mier (1983), Speck (2007)) revealed that -especially in case of concretes beyond C40-theory of plasticity cannot be applied to concrete. The "plasticity" of RC slabs results from the "residual elasticity" of the concrete compression zone of slightly reinforced cross sections: reference shall be made to the limit of mechanical rate of reinforcement at plastic moment redistribution of slab systems, see EC2 (1997) and MC2010 (2012). It is not the concrete material itself, but this plasticity attributed to the concrete compression zone "results" from the difference between the depth of compression zone calculated assuming perfect bond between rebars and concrete and the real behavior of the cracked RC section, provided that low amount, moderate diameter rebars are applied. Incidentally

the USS described as function of the principal stresses is always per se convex.

To d): the triaxial compression curve (or meridian) and the triaxial tension curves are direct "results" of the physical possibilities of the triaxial cells: the stresses in the transversal directions were always identical, which corresponds to the definition of these meridians. In a description of the USS in the coordinate system of principal stresses these meridians are meaningless.

"The experimental studies have shown that ultimate strength of concrete subjected to multiaxial stresses is controlled by the propagation of microcracks. These microcracks are mainly orientated in one (or both) perpendicular direction to the direction of the smallest principal compressive stress (or the two, if their magnitudes are similar) of the largest principal tensile stress."

In the following CEB Bulletin N°156 proposed to apply Ottosen's theory, see later in Chapter 6.

5. VAN MIER

The most important basic statement of van Mier (1984) is: All experiments, also the uniaxial ones, are/must be essentially considered as triaxial.

The strength envelopes for bi- and triaxial experiments by van Mier (see *Fig. 4*) are very transparent and informative. One comment: the validity of the level lines could be questioned: the reciprocity of the strength values $f_u(0;0.1;1)$ and $f_u(0.1;0;1)$ 47 vs. 56 N/mm² is not given. This could be originated from the different concrete batches of the series of relevant specimens. Accordingly, no reliable extrapolations for higher confining stress levels are possible.

Making use of the sense of the strength envelops for bi- and triaxial experiments published by van Mier (see *Fig. 4*)

Figure 4: *Van Mier's (1984) strength-envelopes for the bi- and triaxial experiments. (The tensile axis σ₂ is drawn in a larger scale!)*
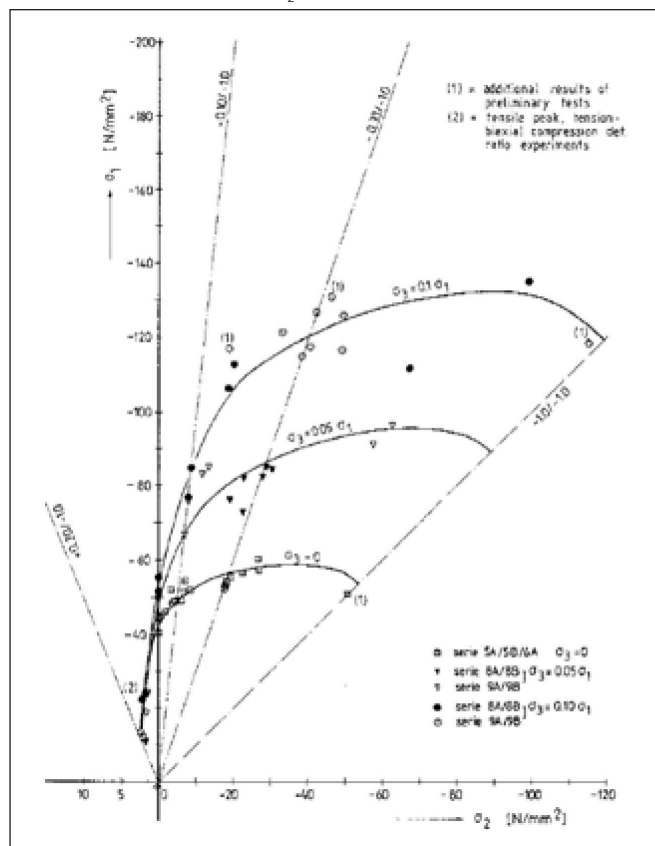
the following dimensioning and control tasks can be followed (the envelopes of the concrete class regarded are known):
- Dimensioning: in case of a given concrete class the strength $f_c^* > f_c'$ shall be reached. $f_c^* > f_c'$ can be achieved along the line $\sigma_3 = f_c^*$ parallel to the σ_2-axis. It must be checked what kind of constrictions perpendicular to the direction of f_c^* (i.e. γ and λ) are given/possible. γ means the steepness of a line in the σ_2-σ_3 plane through the origo; λ is the coordinate of the elevation contour line of the failure surface parallel to the σ_2-σ_3 plane. The intersection of the $\sigma_3 = f_c^*$–line with the line with the steepness of γ in relation to the next two λ-contours along the γ-line gives the necessary rate of confinement in the direction of σ_1.
- Control: concrete class, γ and λ are given. The position of the point along the γ-line corresponding to the λ-ratio shall be determined. The σ_3 ordinate of the intersection is the achievable strength, f_c^*. If the $\sigma_3 \geq f_c^*$ then the strength criterion is fulfilled, otherwise
    o either the rate/s of confinement shall be changed or
    o a higher concrete class shall be chosen.
  As in the tests
- the most possible endeavor was made to have the principal axis parallel with the sides of the cubic specimens,
- the principal components are the most fundamental characteristics of any state of stress
- concrete (and the specimens, too) are never isotropic (at least the direction of compaction have some significant impacts)

hence any transition to the invariants of the deviatoric stress tensor is rather meaningless.

Some discusser called in question the raison d'être of applicability of the loading path and try to consider the problems as uniaxial ones (see the nonlinearity index and the stress-strain behavior splitted in three independent uniaxial characteristics of Ottosen). Van Mier and its *Fig. 4* teach us: any target 3D ultimate strength can be reached following a loading path only: with a 2D loading an increase of 10-20% maximum can be achieved only. Beyond this level loading/ strain restriction in the third direction are necessary to let increase the maximum compressive strength in the 'main/ leading' direction.

6. MC2010

Dealing with the multiaxial states of stress in concrete, MC2010 (2013) treats concrete as frictional material. "The multiaxial criteria should depend not only on shear stresses, but also on the first invariant I_1 of the stress tensor to consider the influence of the hydrostatic pressure on the ductility of the material."

As it could be known since Kármán, concrete is not a "frictional material". Moreover, it has neither a yield function but a failure criterion, nor a flow rule/plastic potential. Especially in case of higher-class concretes after the principal stresses fulfil the failure criteria the concrete loses dramatically its load-bearing capacity, hence its behavior does not allow for a treatment as a "plastic material". Any transformation of the non-linear stress-strain curve into a linear-elastic-"plastic" working diagram (even if with retaining the area under them) falsifies the real character of concrete.

Among several acceptable formulations MC2010 has chosen the constitutive equation of Ottosen as "it is not too difficult to use and agrees well with test data".

The mean value of strength under multiaxial states of stress may be estimated from the failure criterion

$$\alpha \, \frac{J_2}{f_{cm}^2} + \lambda \, \frac{\sqrt{J_2}}{f_{cm}} + \beta \, \frac{I_1}{f_{cm}} - 1 = 0$$

where

$$\lambda = c_1 \cdot \cos\left[\frac{1}{3} \cdot arccos(c_2 \cdot cos3\theta)\right]$$

$$\cos 3\theta = \frac{3\sqrt{3}}{2} \cdot \frac{J_3}{J_2^{3/2}}$$

I_1 is the first invariant of the stress tensor, J_2 and J_3 are the second and third invariants of the stress deviators. In mathematics, an invariant is a property, held by a class of mathematical objects, which remains unchanged when transformations of a certain type are applied to the objects.

(Note: Zhou (1995) recalled that according to the basics of tensor analysis I_1 and $\sqrt{J_2}$ are linearly dependent of each other; hence the formula of Ottosen could be simplified.)

The coefficients α, β, c_1 and c_2 are material parameters which depend of the uniaxial compressive strength f_{cm}, the uniaxial tensile strength f_{ctm}, the biaxial compressive strength f_{c2cm} and the triaxial compressive strength at one point of the compressive meridian ($\sigma_1 = \sigma_2 > \sigma_3$) described by σ_{com} and τ_{com} (these two are the octahedron stresses). In order to determine these coefficients five additional parameters have to be calculated. (Note that f_c and f_{c2} are defined as positive values; all other compressive strengths are negative values.)

It must be diagnosed that the Ottosen model is extremely intransparent.

7. NEW 3D REPRESENTATION OF MULTIAXIAL STRENGTH OF CONCRETE

The most important basic statement: All experiments, also the uniaxial ones, are/must be essentially considered as tri-axial. All possible combinations of stresses which correspond to an ultimate stress state can be expressed in terms of relative principal stresses as an ultimate strength surface (USS).

The stress state in a point is characterized with the three principal strength values. These are ordered as follows: $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Compressive stresses and strains are negative, the tensile ones are positive.

The following notations are introduced:
- Stress state is described in vector form in terms of the principal stress components,
- The loading occurs along a "loading path", i.e. during loading the ratios $\gamma = \sigma_1 / \sigma_3$ and $\lambda = \sigma_2 / \sigma_3$ remain constant, the triade (γ; λ; 1) characterizes a stress-loading path,
- $f_{cu}^* = \sigma_{3u}$ is most negative strength at failure along any stress-loading path.
  Accordingly, $\sigma_{1u} = \gamma \, \sigma_{3u}$ and $\sigma_{2u} = \lambda \, \sigma_{3u}$.

The invariants of the stress and (especially) of the strain tensors remain –as non-transparent and misleading quantities - ignored. Accordingly, the hydrostatic axis, the deviatoric plane, the requirement for the (not realistic) three-fold symmetry and the convexity of the polar figures disappear, too. Similarly, the triaxial compression curve and triaxial tension curve (meridians) in the Rendulic plane vanish.

The hydrostatic stress and strain and the octahedral shear stress and strain hinder the examination of the impact of the three (maximum, intermediate and minimum) principal stresses and strains on the failure surface.

Notions as "equibiaxial" and "equitriaxial" tensile strengths disappear as well: in an equibiaxial tensile test the tensile failure will occur according to the scatter of the tensile strength, independent of each other in the two directions. The same is valid for the equitriaxial tensile tests. The reason is, that –in contrary to the compressive loading, where in transverse direction to the compressive stresses micro- and later macro-cracks occur which influence the actual strength there- the tensile failure occurs in a 'thin' region only perpendicular to the direction of tensile force hence does not influence the tensile strength in the two other directions (See Fictitious Crack Model of Hillerborg et al. (1976)).

The results of the 2D and 3D tests (σ_{3u}) will be displayed in the coordinate system γ, λ as $\sigma_{3u} = f(\gamma, \lambda)$.

Advantages of this display are:
- The direct impact of the maximum and medium stresses, resp. can be perceived.
- As $\gamma \leq 1$, $\lambda \leq 1$ hence the USS has "natural limits" at $\gamma = 1, \lambda = 1$.

It should be recognized that for each concrete class only two failure configurations characterized with the triade (γ; λ; 1) exist:
- the direction of σ_{3u} coincides with the direction of the pouring/compaction,
- it does not coincide.

This means that each failure surface displayed in the (σ_1, σ_2, σ_3) coordinate system is monovalent over the (σ_1, σ_2)-plane. The description using the octahedral stress components suggests an axis-invariance which in the case of concrete (if only because of the direction set by the compaction) might lead to faulty assumptions as due to its production technology concrete is not isotropic. This is even truer in case of fiber reinforced concrete. *Fig. 5* shows the proposed 3D representations of van Mier's results presented in his Fig. 5.8 (1984) and Speck's results with a C80/95 concrete.

One advantage of this type of representation is that in the $\gamma = \sigma_1 / \sigma_3$ direction no special function with regard of a strength under hydrostatic loading conditions must be found. The renunciation of the hydrostatic- and deviator-related representation yields a
- clear and transparent understanding of the influence of the minor (γ)- and intermediate- (λ) stress levels resp.,
- deviating from the compulsory three-fold symmetry with respect to the hydrostatic axis the figures meet the non-isotropic characteristics of the concrete which is the direct consequence of concrete production technology (pouring). It is even more pronounced in case of the fiber-reinforced concretes, which are more and more coming.
- It reveals how misleading is the validation of the Ottosen model using the uniaxial compressive strength (point on the compressive meridian), biaxial compressive strength (point on the tensile meridian), a triaxial compressive strength at one point on the compressive meridian (plus the uniaxial tensile strength). Note: With her very advanced test equipment Speck achieved $\gamma = \lambda = 0.15$ only. Anyway: the strength values along the compressive meridian are of very limited informative value. The maximum strength increase could be anticipated with $\gamma = \lambda = 0.5 \sim 0.6$. The double-curved surface of the failure surface does not allow any reliable extrapolation relying on $\gamma = \lambda = 0$ and $\gamma = \lambda = 0.15$.
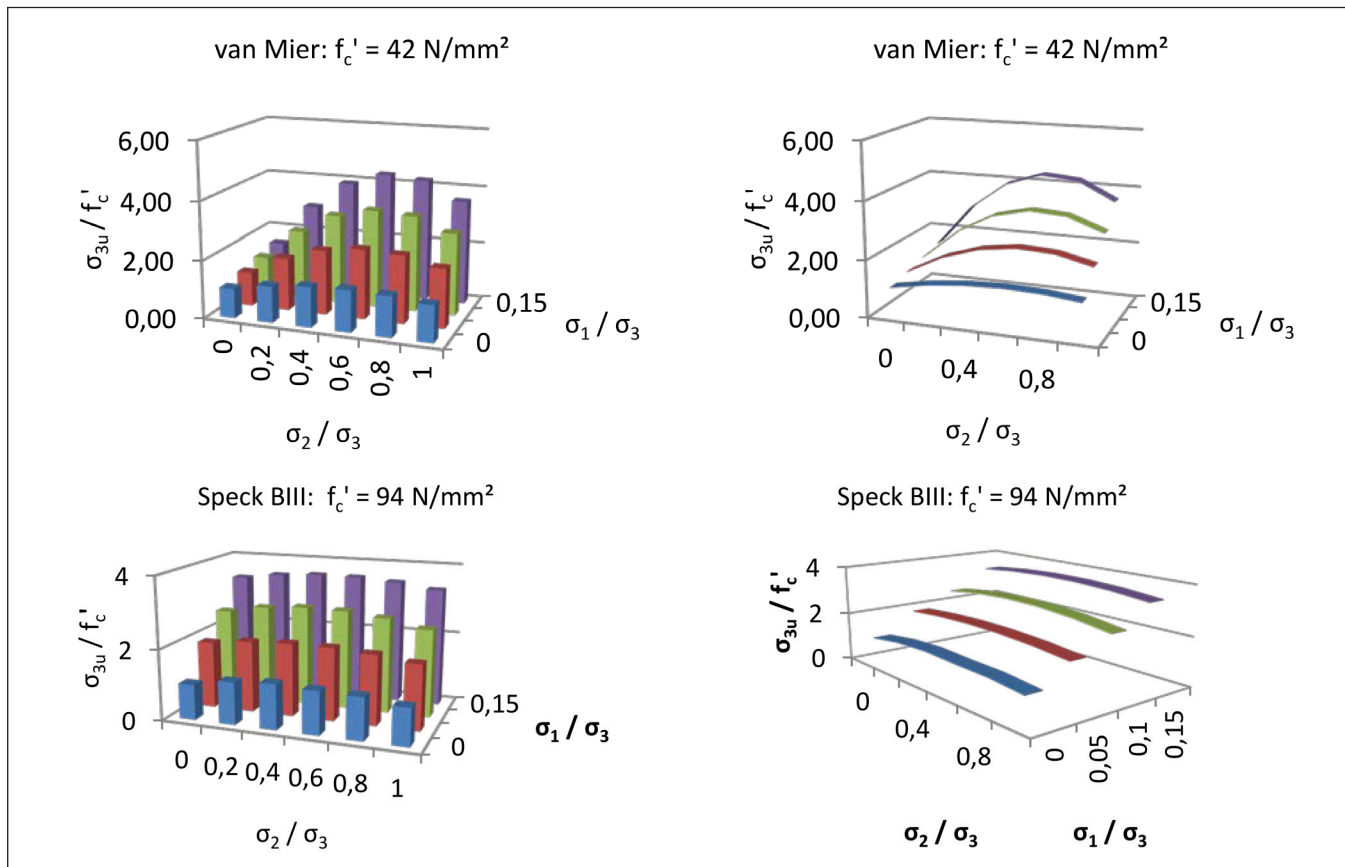
Figure 5: *New type representation of the strength increase of two classes of concrete (van Mier and Speck)*

8. CONCLUSIONS

Kármán's – relatively unknown – theoretical considerations from 1910, related to elasticity and plasticity of ductile and brittle materials and his triaxial loading equipment are presented. The possibilities and restrictions of this equipment determined for 100 years the theoretical and practical works with concrete strength. After a review of some works a new-type 3D representation of triaxial concrete strength is proposed which is transparent and physically sound.

9. NOTATIONS

I_1	first invariant of the stress tensor;
J_2, J_3	second and third invariants of the stress deviators
f_c'	concrete compressive strength
$f_{cu}*$, $f_{ct}*$	ultimate strength (compression and tension resp.) in 2D and/or 3D loading
α, β, c_1 and c_2	material parameters (Ottosen)
$γ = σ_1 / σ_3$	loading parameter
$λ = σ_2 / σ_3$	loading parameter
$σ_1$, $σ_2$, $σ_3$	principal stresses ($σ_1 ≥ σ_2 ≥ σ_3$)
$σ_{c3u}$	ultimate strength measured in test
$σ_{com}$	hydrostatic normal stress
$τ_{com}$	octahedral shear stress
[γ; λ; 1]	stress-loading path
[1;1;$σ_{c3u}$]	ultimate hydrostatic normal strength (cap value)

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Andor Windisch PhD, Prof. h.c. retired as Technical Director of Dywidag-Systems International in Munich, Germany. He made his MSc and PhD at Technical University of Budapest, Hungary, where he served 18 years and is now Honorary Professor. Since 1970 he is member of different commissions of FIP, CEB and **fib**. He is author of more than 180 technical papers. Andor. Windisch@web.de