ON THE EFFECTIVE CONCRETE COMPRESSIVE STRENGTH IN THE THEORY OF PLASTICITY

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Dedicated to Prof. György L. Balázs
for his 65th birthday

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In his fundamental book Nielsen (1984) summarized the basics of application of plastic theories to the design of concrete structures. Since that time extended experimental and theoretical works were carried out. Models based on plastic theories were developed for bending, shear, for beams, plates, etc. The strut-and-tie-, the stress field models and the Modified Compression Field Theory perform further developments of application of the theory of plasticity. Crucial point (the governing material characteristic) of these models is the effective concrete compressive strength. This paper critically reviews its different theoretical origins and despairs of its existence. It reveals that the main source of plastic behavior of structural concrete structures is the reinforcement. In design for shear instead of a preposterous effective concrete strength the effective web thickness and (a maybe slightly reduced) concrete compressive strength should be taken into account. Neither the concrete compressive strength nor any reduced (effective) value of it are applicable as governing material characteristic of any plasticity model for structural concrete structures.

Keywords: theory of plasticity, effective compressive strength, strut-and-tie-model, stress field model, Modified Compression Field Theory

1. INTRODUCTION

In the second half of the twentieth century the application of the theory of plasticity in design of r.c. structures was one of the main developments. The fundamental textbook of Nielsen et al. (3rd edition: 2011) summarized the results so far. In the following decades different models based on plastic theories were developed for bending, shear, for beams, plates, etc. The strut-and-tie-, the stress field models and the Modified Compression Field Theory are to be mentioned here. Crucial point (the governing material characteristic) of these models is the effective concrete compressive strength.

One of the basic assumptions of these models is that at application of the plastic theory to concrete the concrete must become plastic and fail in compression. At application of these models to the results of different test series the concrete in compression failed (numerically) far below the strength value determined on the parallel produced cylinders. The nimbus of the effective compressive strength was born. It didn't bother anyone that for each type of structural element, each type of loading action and each model different efficiency factors has been received. Until today it was not possible to find a generally valid efficiency factor.

The reason is very simple: in case of r.c. material to reach the plastic ultimate limit state both materials do not need to be in a plastic state. The one parametric loading procedure of a r.c. member/structure begins at zero. Due to the limited tensile strength of concrete first 'event' fulfilling the lower limit theorem is the cracking of the member. In the cracked cross sections the reinforcement crossing the cracks ensures the tensile forces needed for the equilibrium. Depending on the applied mechanical rate of reinforcement:

a) one or both bands of reinforcement (under-reinforcement) begin to yield along the crack or

b) in case of over-reinforcement, the concrete indeed fails.

At many test series yielding of the reinforcement reduces the stiffness of the member/structure to such an extent that the hydraulic loading system fails to overcome the deformations: the load drops dramatically, the test will be stopped. In case a) the failure load must not be related to the concrete, otherwise crazy effective concrete compressive strength values will be generated.

In this paper the history of the origin of the different effective concrete compressive strength values is elucidated.

2. THE PLASTIC CONCRETE THEORY OF NIELSEN

In their fundamental book of Nielsen et al. (2011) the main target of the chapter dealing with the yield conditions for concrete is to persuade the reader that concrete is a Modified Coulomb Material and mostly fail at relative low $(\sigma < f_c')$ strength values. *Fig. 1* taken of Nielsen should prove the existence of the effective compressive strength, i.e. existing cracks reduce the effective compressive strength of the stress field.

The Modified Coulomb material (*Fig. 2*) fails either in uniaxial tension with separation or in (uni- and biaxial) compression undergoing a sliding failure. *Fig. 1 a)* is correct. The specimen (prism or cylinder) fails in the testing machine where the friction along the loading equipment and specimen is eliminated due to longitudinal (tensile) cracks (separation failure). *Fig. 1b)*: inclined cracks never develop in a specimen under pure compression. This crack should have occurred due to another state of stress before this loading producing the compressive principal and tensile stresses to become active. Concrete react to principal stresses only. (This is where the

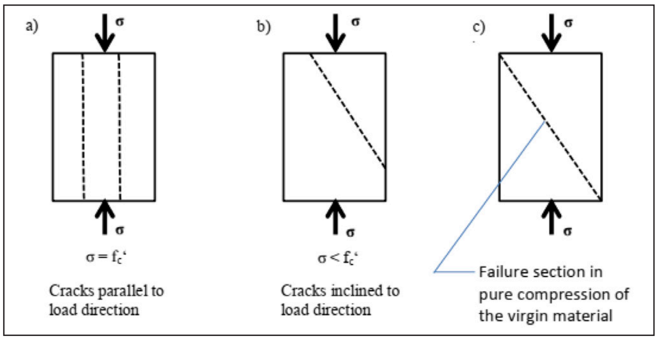


Fig. 1: Influence of cracks on the compressive strength acc. to Nielsen

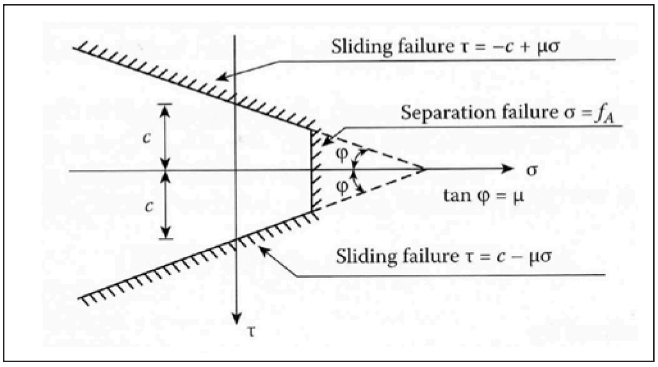


Fig. 2: Modified Coulomb Failure Criterion

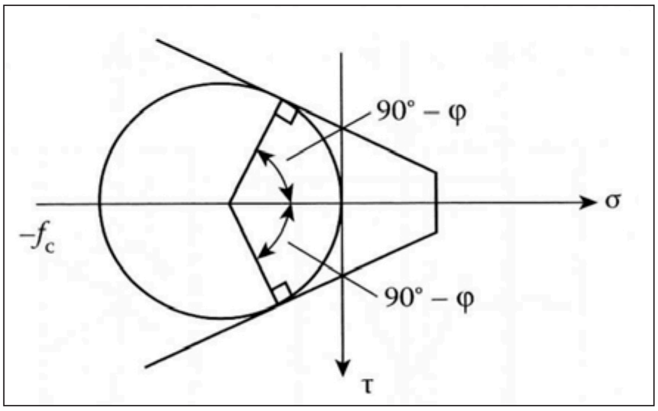


Fig. 3: Mohr's circle of uniaxial compression

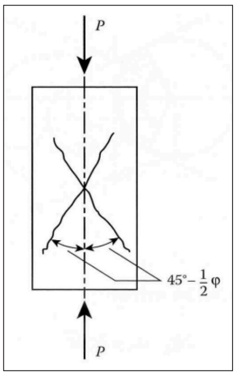


Fig. 4: Failure sections at pure compression acc. to Nielsen

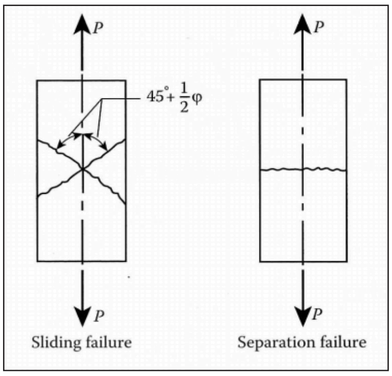


Fig. 5: Failure sections at pure tension acc. to Nielsen compression

distraction of the reader begins!) *Fig. 1c)*: this type of failure never develops. It is in clear contradiction with the correct failure pattern shown in *Figure 1a)*! What should mean the adjective 'virgin'? Important to note: plastic theory is valid in case of one parametric loading only!!! Accordingly the cracks and 'failure sections' due to earlier loading scenarios cannot be considered acc. to the theory of plasticity!!

As a matter of fact from here on, the whole 'theory' of Nielsen is invalid and irrelevant.

Nielsen et al. present the Modified Coulomb Failure Criterion as valid for concrete, see *Fig. 2*. Important characteristics in the σ, τ-coordinate system are the two types of failure: separation and sliding, the τ = c (cohesion) section along the τ-axis and the inclination of the line characterizing the sliding failure, φ. We must note the following:

- the shear stress is a calculation auxiliary quantity only. It 'owes' its existence to the Cartesian global coordinate system. Turning the axis of the coordinate system the stress components (the shear stress, too) continuously change. Not a single stress component can be treated as 'shear strength'.
- The Coulomb Failure Criterion (CFC) is valid in case of already existing sliding surfaces only!! Glue the brick in the well-known brick-on-slope-test to the slope and try to get the brick to slide. You will fail. Q.e.d.

In *Fig. 3* Nielsen shows the Mohr's circle of the uniaxial compression and declares that as the Mohr's circle obviously touches the line of the sliding failure in case of uniaxial loading sliding failure might occur, see *Fig. 4*, where the inclination of the sliding (failure) surfaces are 45° - φ/2 inclined to the loading direction. The following objections must immediately be raised:

- Do not forget that Mohr's circle is a two-dimensional graphical representation of the transformation law for the Cauchy stress tensor, i.e. formulas containing $\sin^2$, $\cos^2$ and sin·cos terms.
- All points of the Mohr's circle are equivalent, i.e. the two 'touching points' are not of distinguished rank at all
- As mentioned before, the clear failure pattern of test specimens in pure compression is as shown in *Fig. 1a*. Inclined failure surfaces develop due to not eliminated friction between loading plate and test specimen. Changing the rate of friction the measured strength and the direction of the actual principal stress, i.e. of the actual failure surface, change too.

Fig. 5 shows the failure sections at pure tension acc. to Nielsen. The separation failure is correct. The single raison d'être of the sliding failure pattern at pure tension is that it is possible to draw on a piece of paper a tangent to the Mohr's circle characterizing the pure tension. Under no circumstances can such a fracture pattern occur in the practice at pure tension.

Pure shear loading results in a principal tension and a principal compression stress of the same size and under 45° directions. As the tensile strength of concrete is very limited the loading procedure finishes with a tensile separation failure.

Let us take a closer look at the Modified Coulomb Failure Criterion: for its definition three data are necessary: c, μ or φ and the tensile strength. In the best case, we only have two material characteristics at our disposal: the compression and the tensile strengths. Even if the Modified Coulomb Failure Criterion would be valid for concrete how could we deduce the necessary coefficients? As mentioned above: pure shear loading results in principle tension. Accordingly the 'cohesion' c must be equal with the concrete tensile strength. Let›s calm down. Modified Coulomb Failure Criterion does not apply to concrete at all. Concrete obeys only principal stresses and performs only separation failures.

Thereafter follows the big scientific obfuscation: the gentle reader should distinguish the strength of

- "Cement mortar (isotropic plastic material with the friction angle φ = 0)

- Two-phase material: cement paste and aggregate particles. It is well known that cement paste behaves in a rather brittle way in uniaxial tests. It is explained by the presence of hard unhydrated cement particles. Sliding failure may be prevented by the hard particles leading to the formation of yield lines in front of these particles. … There will also be some resistance from other parts of the yield line, dependent on the amount of microcracking and the amount of load induced cracking. Acc. to the model, the failure in cement paste is always ductile for compression stress fields and the apparent brittleness in cases of small confinement is due to the relatively short lengths of the yield lines formed in front of the hard particles…. You need not try to understand this confusion.
- Structural concrete strength ("Unfortunately, the strength of concrete we observe when testing a structure is usually very different from the strength measured on standard laboratory specimens. The main reason is that the concrete is cracked, and cracking reduces the strength.

The most important consequence of this fact regarding the application of plastic theory is that the strength parameters, which we have to insert into the theoretical solutions, normally are lower than the standard values. We call the strength values to be inserted the effective strength. The effective concrete compressive strength f_{cef} is, defined by

$$f_{cef} = \upsilon f'_c$$

where $\upsilon \leq 1$ is called the effectiveness factor for the compressive strength."

That's all.

"The strength reduction due to cracking might be subdivided into (a) strength reduction due to microcracking present even before any load is applied, (b) strength reduction due to load-induced microcracking and finally (c) strength reduction due to macrocracking. While the microcracking present before loading may be assumed to lead to an isotropic material, load-induced microcracking and macrocracking will cause anisotropy, i.e. the strength parameters, for instance the compressive strength, will vary with direction. Strictly speaking, cracked concrete should be treated as an anisotropic material.

However, this cannot be done in a fully rational way… We must content ourselves with a more primitive approach. It consists of either considering cracked concrete to be isotropic with the effective strength parameters or by dealing with the strength parameters only in certain selected direction depending on the crack system."

Not a single word of any scientific justification can be found… First the cement mortar is an isotropic plastic material thereafter rather brittle … You need not understand this confusion.

This order to present a theoretically founded reason of the strength reduction *Fig. 6* is presented: "two parallel macrocracks, which are crossed by a reinforcement bar perpendicular to the cracks, the bar is stressed in tension. Let us assume that some kind of microcracking is spreading out from the macrocracks when the reinforcement stress is increased. The microcracks are shown schematically in the figure. The real microcracking is of course much more complicated." Note: plastic theory is valid in case of one parametric loading only, i.e. the steel stress, the alleged microcracks and the compressive stress between the two macrocracks increase parallel. In addition, the compressive stress prestresses the 'strut', i.e. the impressive and convincing microcracks occur

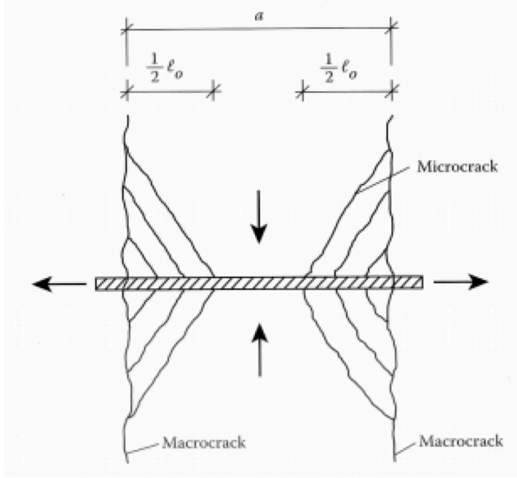


Fig. 6: 'Formation of microcracks between two macrocracks' acc. to Nielsen (1984)

in a completely different manner, if at all. Do not forget, that microcracking is a natural phenomenon parallel with the compressive loading even during the material testing. The proposed efficiency factor has no theoretical background it is an empirical coefficient which should adjust the shortages of the shear model choosing the direction of the compression field arbitrarily across the shear cracks.

Recent studies revealed that aggregate interlock has negligible contribution to the shear failure load accordingly the inclined compression fields do not cross the shear cracks. Thus, there is no need for the efficiency factor any more.

"We have now finished our review of the rather incomplete knowledge about the effective strength of concrete. It appears that an accurate description of the real behavior of concrete is not possible by simple means. In the main part of this book, we will therefore take an engineering approach to the problem.

The main line will be to develop solutions using plastic theory based on the modified Coulomb failure conditions. These solutions are then modified by the introduction of effective strength parameters determined on basis of the tests available. The physics of the problems may then be well hidden, but it is believed that such an approach will be the most useful for the engineering profession at the present stage of development. By any measure it will be far more useful than a completely empirical approach, which is still dominating many areas of the concrete field."

Heureka.

From now on the applier of the Nielsen-type plastic theory have a multiplier $0 < \nu < 1$, through its application they can adjust their results of calculation to any test results.

3. SHEAR FAILURE OF BEAMS WITHOUT SHEAR REINFORCEMENT

Fig. 7 shows the final crack pattern of a simply supported beam without shear reinforcement loaded with two symmetrical concentrated forces acc. to Nielsen et al. (2011). "The curved cracks in the shear zone are running from the tensile face toward the nearest force when the load is increased. One of these cracks leads to failure because of the low sliding resistance along the crack. The final failure will be a sliding failure along OA and BC and a separation failure along AB,

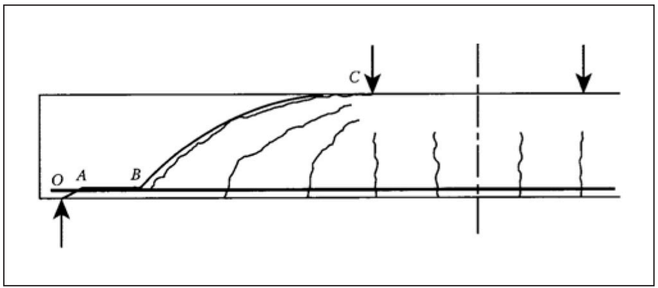


Fig. 7: Failure of a beam without shear reinforcement

which is situated just above the longitudinal reinforcement. The yield line along the crack will be more dangerous than a yield line through concrete without microcracks."

This failure pattern needs some comments: The upper part of the flexural-shear crack BC should consist of two parts: section below the neutral axis and along the compression zone, resp. otherwise no flexural equilibrium exist. Moreover, instead of sliding the parts left and right from ABC, resp, rotate relative to each other around the point where the shear crack crosses the neutral axis. Interestingly the compressive forces/stress fields are not mentioned.

Using this unrealistic failure model we arrive at a nonrealistic/not existing material characteristic: the shear strength of concrete.

4. EFFECTIVE COMPRESSIVE STRENGTH OF THIN BEAM WEBS IN SHEAR

Perform the following thought experiment:

Given is a series of simple span beams with four-point loading, their shear slenderness is three. All data are identical (including the diameter, distance and concrete cover of the stirrups), only the web width decreases from beam to beam). The failure in flexure and shear is triggered by yielding of the longitudinal and/or the shear reinforcement. (Often the stirrups spall off the concrete cover nevertheless the stresses in the 'concrete struts' are far from a failure due to compression. The failure occurs when a sliding surface through the compression zone develops and the flexural reinforcement spalls off the concrete cover: a kinematic flexural-shear hinge develops: displacements in the structure progressively increase: failure is attested.) Consider now the development of the inclined compressive stresses: in the beam with rectangular cross section it is small and increases with decreasing web thickness. The stirrups might spall off the concrete cover and at a certain web thickness the 'concrete struts' between the stirrups fail in compression at a compressive stress near to the original compressive strength. Only at this beam occur the shear failure due to both, yielding of steel and exhaustion of load-bearing capacity of concrete. When the advocates of the plastic theory evaluate the results of this last test then the inclined compressive force is related to the whole width of the web, i.e. a pronounced low compressive strength is 'produced numerically', a heavy softening of concrete in compression is attested.

We remind you that in the Codes (e.g. MC2010, 2013, Ch. 7.3.3.3) at calculation of the design shear resistance attributed to the concrete "in case of prestressing tendons in the web with duct diameters $Ø_D \geq b_w/8$, the nominal value of the web width

$$b_{w,nom} = b_w - k_D \sum Ø_D$$

Values of k_D depend on the material of the duct and whether it is grouted or not. Suggested values for design are:
- grouted steel duct. $k_D = 0.5$
- grouted plastic duct: $k_D = 0.8$
- ungrouted duct: $k_D = 1.2$.

Factor k_D may be reduced in presence of reinforcement transverse to the plane of the web."

As a matter of fact all the rebars in the web: the longitudinal as well as the transverse ones (stirrups) are disturbing elements in the compressive stress field, too. After formation of the flexural-shear cracks the rebars crossing the cracks perform a relative displacement (slip) related to the concrete, i.e. the rebars are in a channel, similar to the ducts, accordingly at the calculation of the nominal value of web width similar reductions should be taken into account. Doing so the fairy tale on the softened concrete strength could be eliminated.

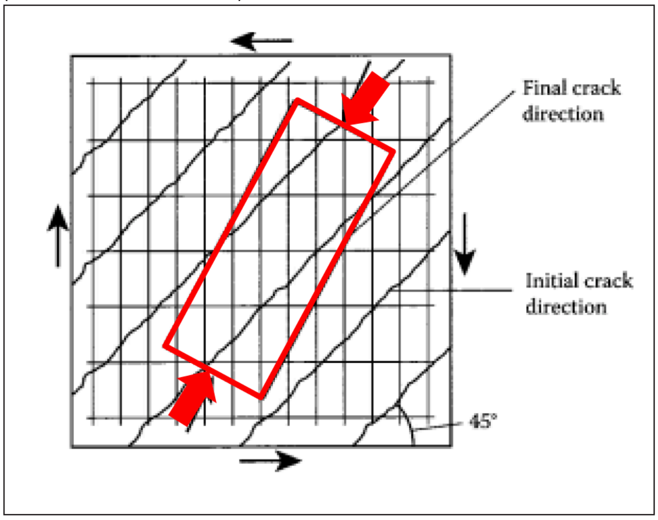
The softened (effective) concrete strength is currently like a dominant religion: researcher who measure contradictory results are ashamed to publish them, they would be considered as heretics.

5. SLIDING FAILURE OF ORTHOTROPIC PANELS

The panel shown in *Fig. 8* is reinforced with different ratios in perpendicular directions. The initial cracks are formed under 45° with the sections with pure shear and their directions are roughly independent of the reinforcement (so far is Nielsen right)."However, if the reinforcement ratios in the two directions are different, the final crack direction will be different from the initial one. This means that the final compression direction in the concrete might be as shown in *Fig. 8*. It follows that compressive failure may take place by sliding failure along the initial cracks for a very low compressive stress compared to the compressive strength of the virgin material. Such a reduced compressive strength has been measured by Vecchio and Collins and they also measured the strains."

Author's comments (Windisch, 2000):
- Nielsen's statements about the initial cracks are correct. The parts of the concrete panel perform a separation on both sides of the tensile cracks perpendicular to it
- the red rectangle with its axial loading and the 'final crack

Fig. 8: Orthotropic panel with initial and differing final crack direction (acc.to Nielsen et al. 2011)



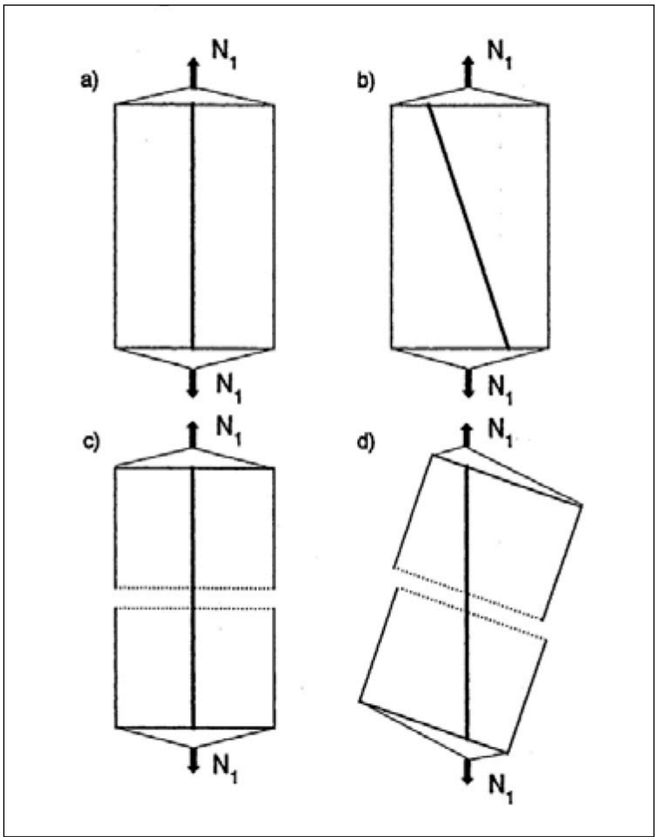


Fig. 9: *Displacements of a membrane element under uniaxial tension*

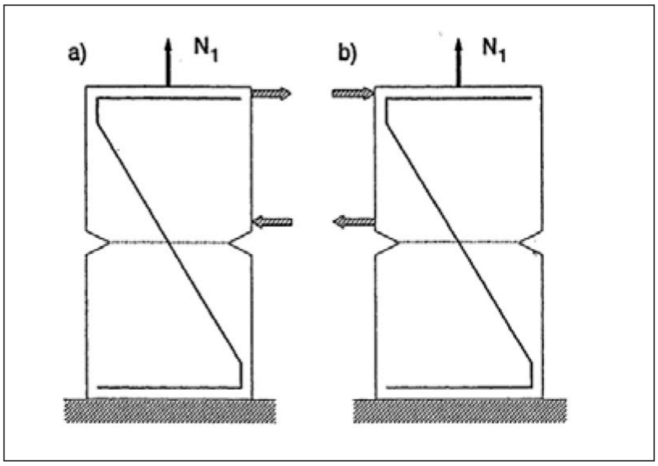


Fig. 10: *Supplementary forces depend on the boundary conditions*

directions' emerge on the drawing board only, never as a consequence of the initial 'pure shear' of a concrete panel

- the panel's behavior after development of the initial cracks cannot be predicted without the knowledge of the boundary conditions.

Fig. 9 shows displacements of two membrane elements under uniaxial tension: the element a) is reinforced in the direction of the tensile force (or with equal rates in two directions symmetrical to the longitudinal axis), the element b) in a different direction (or with equal rates in two directions not symmetrical to the longitudinal axis or with different rates in arbitrary directions). The sketches c) and d) show the corresponding free sliding which are hindered i) by the dowel action of the rebars and ii) through the boundary conditions. Supplementary forces develop, their size and sign (compression or tension) depends on the exact boundary conditions (*Fig. 10*).

In *Fig. 8* the direction of the imagined 'final' compressive force is equated with final (new) cracks. As well known the tensile crack unloads the concrete on its both sides, hence the cracks cannot rotate neither under the influence of the initial pure shear nor under the unknown supplementary forces. From now on, all further derivations are fiction contrary to the basic properties of reinforced concrete. The resulting discrepancy between the calculation and the experimental results is intended to correct the effective compressive strength of the concrete (also arbitrarily declared). It is interesting to follow what kind of pull-ups are done 'to bring' concrete in 'failure' condition.

Vecchio's and Collins's Modified Compression Field Theory will be discussed in Chapter 7.

6. YIELD CONDITIONS FOR ORTHOGONALLY REINFORCED DISKS

In the following Nielsen's deduction (2011) for the case of pure shear loading is quoted and commented.

"The disk is loaded in pure shear in the x, y-system. For the concrete part of the equilibrating composite material system a concrete stress field characterized by the principal stresses

$$\sigma_{c1} = 0, \sigma_{c2} = - \sigma_c, \tag{1}$$

with the second principal axis forming an angle α to the x-axis are supposed."

Note:

- Pure shear loading τ in the x, y-system means principal stresses $\sigma_1 = \tau$ and $\sigma_2 = -\tau$, the principal axes (we denote them I and II, resp. in order to distinguish them from the 'principal axes' as defined by Nielsen) are inclined at ±45° to the x-axis,
- the assumption $\sigma_{c1} = 0$ means that in the direction of the first principal axis the equilibrium is balanced by the tensile forces in the rebars only, i.e. the concrete is cracked along the border of the concrete stress field.

From the compressive stress $-\sigma_c$ in the stress field Nielsen obtains (compressive) stresses

$$\sigma_{cx} = - \sigma_c \cos^2\alpha, \quad \sigma_{cy} = - \sigma_c \sin^2\alpha. \tag{2}$$

Defining

$$A_{sy} / A_{sx} = \mu \tag{3}$$

He gets in the x- and y-direction

$$A_{sx} f_{Yx} = t\, \sigma_c \cos^2\alpha \quad \text{and} \quad A_{sy} f_{Yy} = t\, \sigma_c \sin^2\alpha \tag{4}$$

Note: we must fulfil the equilibrium condition in the crack along the border of the concrete stress field as Nielsen simply just doesn't care of it.

$$A_{sx} f_{Yx} \cos^2 45° + A_{sy} f_{Yy} \sin^2 45° = 0.5\, (A_{sx} f_{Yx} + A_{sy} f_{Yy})\, \tau \tag{5}$$

This means that at design we can choose the amounts and yield strengths of the orthogonal reinforcement nevertheless two points must be considered:

- parallel to the crack components of the tensile forces in the orthogonal reinforcement develop which shift the two parts of the disk separated by the 45° inclined crack. Depending on the support conditions of the disk (of the test specimen) which hinder the shifting, different supplementary forces (compressive and/or tensile stresses) can develop in the disk.

- it must be emphasized that when the weaker band of reinforcement begins to yield then the deformation of the disk might become partly uncontrolled and it could occur that the loading procedure is finished before the stronger band of reinforcement yields. We recognize that the concrete strength does not influence the behavior of the disk at all, hence the maximum concrete stress achieved during the experiment cannot be attributed to failure of concrete: in any case the concrete did not experience any ‘softening’.

“From the two equations (3) and (4) Nielsen compiles a definition for the angle α

$$\tan^2\alpha = \mu, \tag{6}$$

he gets

$$\sigma_c = \Phi_x\,(1 + \mu)\,f_c \tag{7}$$

he pretends that the compressive stress in the stress field is function of the concrete strength nevertheless replacing the definitions of Φ_x and μ we get

$$\sigma_c = (A_{sx} + A_{sy})\,f_Y / t \tag{8}$$

i.e. the compression stress in the stress field is completely independent from the concrete class, so far the plastic theory for concrete by Nielsen…

“Having

$$|\tau_{cxy}| = \tfrac{1}{2}\,\sigma_c \sin 2\alpha \tag{9}$$

The shear strength of concrete is

$$f_v = \frac{\sqrt{A_{sx} \cdot A_{sy}}}{t}\, f_Y \qquad (10)”$$

Once more there is no sign of dependence on the concrete class.

Note: Eqs. 3, 6, 8 and 10 are valid when the yield strength of A_{sx} and A_{sy} are identical, otherwise f_{sxY} and f_{syY} must be incorporated in the relevant terms.

“This example of using our assumptions clearly shows the importance of the assumption that the tensile strength is zero. We find that in all sections parallel to the η-axis the stress of the concrete is zero, which can be said to correspond to a continuous distribution of “cracks”. In reality, the initial cracks will be in sections under 45° with the coordinate axes x and y. Tensile stresses in these sections would appear before cracking, and the elongations in the x- and y-axes would be zero (i.e., the bars would not come into action). As soon as cracking appears, however, the bars get into action and the uniaxial concrete compression stress rotates to a new direction if $\mu \neq 1$. New cracks may be formed. Thus we may get crack sliding in the initial cracks.”

Could somebody explain how and for what reason cracks in new direction could/should appear? Moreover, having continuous (smeared) cracks the reinforcement will never achieve yielding. Nothing is right/correct here!

All stress field models, STM and MCFT are based on this false assumption.

7. MODIFIED COMPRESSION FIELD THEORY

The basic element of the Modified Compression Field Theory is the unidirectional or orthogonal reinforced membrane panel which can model the web of beams or a box formed from four as members loaded in torsion.

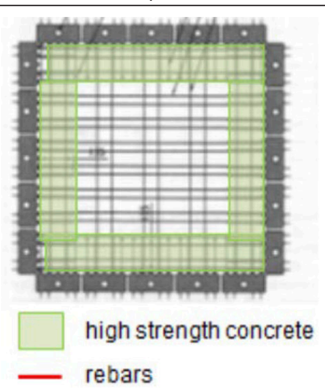
Vecchio and Collins (1982, 1986) developed a panel tester where 80cm x 80cm big panels were loaded in pure shear or combined loading. For more details look also to Windisch (2010).

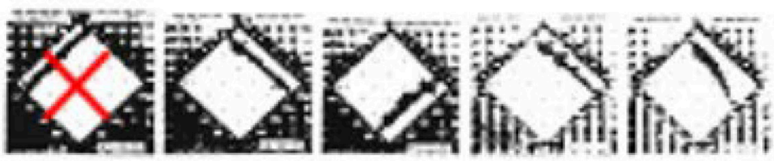
For most specimens, two separate concrete mixes were used. A relative strong concrete was cast in a 100 mm band around the perimeter; a weaker mix was cast in the central regions of the panels”: even if the mechanical strain measurements were taken only in the central regions, the stronger boundary concrete unloads the weaker central region thus “pollutes” the results.

The reinforcing meshes in the test panels were constructed of smooth wires welded into an orthogonal grid; typically at 50 mm (2 inches) centres. The smooth bars provide a completely different performance in the concrete panel as deformed rebars, hence the results and conclusions, even if they would be correct, cannot be generalized to panels reinforced with deformed rebars.

MCFT treats “the reinforcement and the cracked concrete separately. The two materials are ‘tied together’ by the compatibility requirement that the strain in the reinforcement and in the concrete equal those in the panel (*Fig. 11*). It should be noted that the concrete component is not equivalent to an unreinforced concrete element. Rather, it is the concrete in a reinforced concrete element minus the reinforcement.”

Fig. 11: *Toronto-testpanel and identical failure patterns of very differently loaded panels*





	PV24	PV25	PV23	PV27	PV28
f_n/v	-0.83	-0.69	-0.39	0	0.32
v_u	7.94	9.12	8.87	6.35	5.80

"Constitutive relationships are required to link average stresses to average strains for both the reinforcement and the concrete. These average stress-average strain relations may differ significantly from the usual local stress-local strain relations determined from standard material tests. Furthermore, the average stress-average strain relationships for the reinforcement and for the concrete will not be completely independent of each other, although this will be assumed to maintain the simplicity of the model. The axial stress in the reinforcement will be assumed to depend only on one strain parameter: the axial average strain in the reinforcement. In relation of axial stress to axial strain, the usual bilinear uniaxial stress-strain relationship was adapted."

Vecchio and Collins refer clearly and correctly: "the stress" and strain formulations deal with average values and do not give information regarding local variations. At a crack, the tensile stresses in the reinforcement will be higher than average, while midway between cracks they will be lower than average. The concrete tensile stresses, on the other hand, will be zero at a crack and higher than average midway between cracks. These local variations are important because the ultimate capacity of a biaxially stressed element may be governed by the reinforcement's ability to transmit tension across the cracks."

These local variations are in fact very important, as it will be shown in this paper. It is out of all reason why Vecchio and Collins did not follow this way. During the iteration procedure Vecchio and Collins continuously refer to the necessary control of the steel stress level in the reinforcement, nevertheless, it is not told, what should be done in such cases.

Moreover, at each and every test panel which did not fail at an early stage due to some discrepancies, the failure was always induced by yielding of the weaker i.e. transverse reinforcement: this means that the achieved principal compressive stress depends on the characteristics of the reinforcement which is not reflected in the $\varepsilon_{dt}/\varepsilon_d$ values at all. Further on, if the transverse tensile strain should have a degrading effect then in case of deformed bars this effect must be completely different (even stronger) as deduced in the report.

The principal compressive stresses taken as the basis for the well-known formula:

$$\frac{f_p}{f_c'} = \frac{1}{0.85 + 0.27\,\varepsilon_{dt}/\varepsilon_d} \tag{11}$$

were derived directly from the average steel stresses (*Fig. 12*). Here ε_d is the principal compressive strain whereas ε_{dt} is the principal tensile strain, both in the diagonally cracked concrete. The additional compressive forces developing due to the fixed links were not considered these are those deviations which gave the impression to Vecchio and Collins of the softening concrete under the influence of $\varepsilon_{dt}/\varepsilon_d$.

It is very important to emphasize that not a single of the panel failure patterns showed a compression character. In *Fig. 11* a series of panels with systematically changing loading (from shear + compression to shear + tension) is shown: the failures occurred systematically at the junction of central panel concrete to the border concrete.

Fig. 13 shows the comparison of calculated vs. measured failure loads of the same panels as in *Figure 12* as calculated by Windisch (2000). The failures were obviously triggered by yielding of the reinforcement.

Fig. 14 reveals that the measured yield loads are between the double yield load of the weaker band of reinforcement and sum of the yield loads of the weaker and stronger reinforcement. The concrete strength has no input at all.

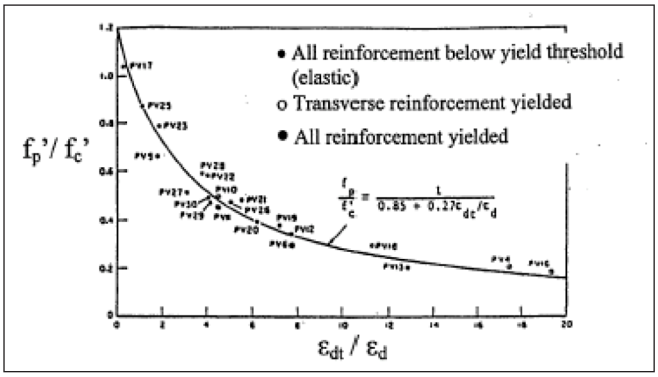


Fig. 12: Compressive strength softening acc. to Vecchio and Collins (1982)

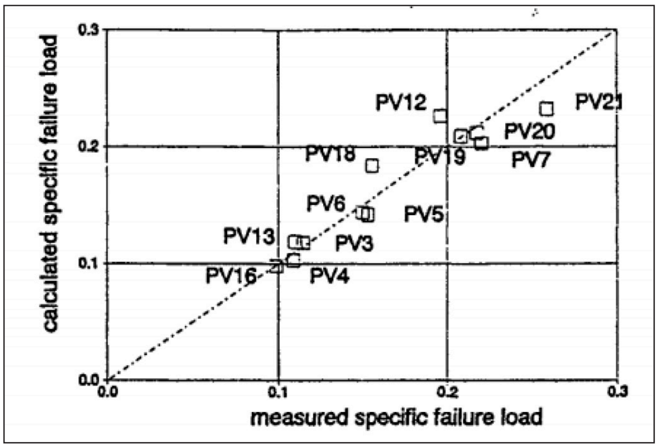


Fig. 13: Comparison of calculated vs. measured failure loads (Windisch, 2000)

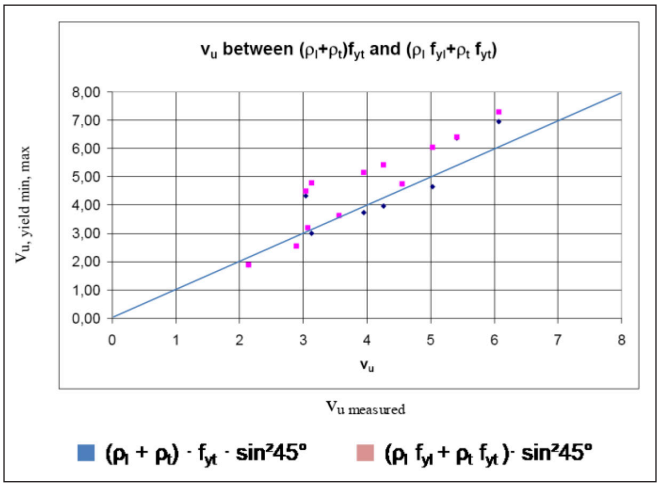


Fig. 14: Comparison of lower and upper yield loads with the measured yield loads

8. STRUT-AND-TIE MODEL

The strut-and-tie model (STM) was proposed by Schlaich et al. (1984, 1987). The basic concept of the generalized truss model states that the flow of the force within a reinforced concrete structure is the same as the flow of the force within a truss. This truss model comprises straight compressive stress fields and straight tension ties. The point at which at least three forces (lines) gather for force equilibrium is called a node. The concrete area at the node position that allows the strut and tie forces to be transmitted through the node is called the nodal zone.

Structural members can be divided into B-regions (Beam or Bernoulli region), where linear strain and beam theory are applied, and D-regions (Disturbed or Discontinuity region) where beam theory is not applied due to applied concentrated loads or discontinuous cross-sections.

Design methods for strut-and-tie models define the nominal compressive strength of a strut as the product of the effective compressive strength of the concrete and the cross-sectional area at one end of the strut. The effective compressive strength is defined e.g. in Section A.3.2 in ACI 318 (2014) as:

$$f_{ce} = 0.85\ \beta_s\ f_c'$$

where f_c' is the specified compressive strength of the concrete and β_s is a factor "to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in the strut."

A STM has two free parameters in order to adapt the model to the measured load bearing capacity: the inclination of the struts and the effective compressive strength of concrete. In the absence of well-founded theories the researcher tried to find the effective compressive strength of concrete evaluating huge data banks.

The simplest and most often tested structural element is the deep beam. Strut capacity was found to be a function of the effective compressive strength of concrete and is affected by *a*/*d* shear slenderness, concrete's strength, load duration effect, transverse tensile strain, and cracking. Normalizing the test results with a/d $\sqrt{f_c'}$ Brown et al. (2008) found efficiency factors, υ between 0.2 and 2.5. The simple reason is that r.c. structures and deep beams, too, don't work like trusses.

Moreover, besides the prismatic strut the bottle-shaped strut was introduced. Drawing the belly stress fields it was inevitable that their curved borderlines crossed cracks. There was great fear and joy that they found a reason for strength reduction that seemed realistic.

Already in 2010 Windisch pointed out that the bottle shape and the related strength reduction contradict the theory of plasticity.

In a test series Pujol et al. (2011) proved the unsustainability of the strength reduction, nevertheless due to collegiality (?) they did not dare to declare that the emperor was naked.

„The reported results show no support for attributing the reduction in the allowable compressive strength at the ends of the „bottle-shaped" strut to the shape of the strut. This does not mean that the limit set by ACI 318-11 for allowable strength of bottle-shaped struts is incorrect. It simply means that the rationalization used for explaining the reduction is incorrect and unnecessary.

In 1932, Hardy Cross wrote, „All analyses are based on some assumptions which are not quite in accordance with the facts. From this, however, it does not follow that the conclusions of the analysis are not very close to the facts."

The fact that the rationale for the strut-and-tie method is incorrect does not necessary mean that the results of the procedure is incorrect. However, it does mean that the method could be explained in a simpler, if arbitrary, fashion. There is little reason for representing the strut-and-tie method in analytical abstractions."

Let's make it clear: at design of deep beams the amount of flexural reinforcement should be properly determined (the inner lever arm cannot be taken freely), the rebars should be properly anchored, the loading- and the support plates properly dimensioned. No $\upsilon < 1.0$ reduction factors need to be taken into account, moreover the advantages of partial area loading can be considered.

In 1991 Windisch presented the Strut-Crack-and-Tie Model where the material parameter is the effective steel strength, which depends on the angle between the crack and the reinforcing bar crossing this crack. It was shown that the concrete compressive strength is not the governing factor of the load bearing capacity of structural concrete structures determined acc. to the theory of plasticity hence the effective compressive strength cannot be chosen as fundamental material model characteristic.

9. CONLUSIONS

Theory of plasticity is valid in case of one parametric loading only. It is necessary to accept that when applying the theory of plasticity in the case of reinforced concrete structures in ULS, the concrete does not necessarily have to be in a plastic state (satisfying any fracture condition).
The effective concrete compressive strength, $f_{c,ef}$:

$$f_{c,ef} = \nu\ f_c$$

where $\nu \leq 1$ is called effectiveness factor is the result of a theoretically flawed model idea.

The concrete doesnot obey the Modified-Coulomb-Failure Criterion.

The sliding part of the Coulomb-Failure Criterion is valid in case of already existing surfaces only.

Concrete has no shear strength. It fails due to principal stresses only. The failure occurs by separation along planes parallel to the direction of the principal compressive stress.

Yielding of a r.c. panel occurs when one or both bands of the reinforcement reach their yield strength. Depending on the direction and relativ amount of reinforcement at yielding besides crack opening shifting can occur which will be hindered through the boundary conditions supporting the panel. These let develop secondary stresses in the panel which might be tensile or compressive stresses. Anyway thereafter a new chapter of application of theory of plasticity begins, where the precracked r.c. panel has completely different material characteristics compared to the original panel. Note: the boundary conditions are relevant parts of the history. The concrete compressive strength is not the governing factor of the load bearing capacity of structural concrete structures determined acc. to the theory of plasticity, hence the effective compressive strength cannot be chosen as fundamental material model characteristic.

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