RELIABILITY ASPECTS OF DESIGN OF COMBINED PILED-RAFT FOUNDATIONS (CPRF)

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SUMMARY

The reliability aspects of structural behaviour of CPRF are investigated. The problem is connected with the stochastic model of soil properties. In this approach the influence of autocorrelation of soil parameters is considered. The calculations are made by means of the First Order Reliability Method (FORM) according to Level II of the reliability analysis. The first results are obtained.

Keywords: safety, safety index, reliability, combined piled-raft foundation, soil

1. INTRODUCTION

Standards and design rules for Combined Piled-Raft Foundations (short CPRF) are not available up to now. But the problem of design of Combined Piled-Raft Foundations becomes more and more important in the last years, when some skyscrapers were built in Germany. Most of them are located in Frankfurt on Main.

The CPRF is a good economic decision for high rise buildings because both the bearing capacity of the raft and the bearing capacity of the piles are completely used.

The CPRF acts as a composite construction consisting of the three bearing elements: piles, raft and subsoil. In comparison with the conventional foundations design the CPRF leads to a totally new dimension of the subsoil-structure interaction because of the new design philosophy using the piles up to their ultimate bearing capacity concerning the soil-pile interaction. This leads to an extremely economical Foundations with rather low settlements, if the stiffness of the soil is increasing with depth (Katzenbach, 1993).

The purpose of investigation is to elaborate the safety concept for the CPRF, which should ensure the same reliability level as this of the piled and raft foundations in the current codes.

The task is divided in 2 phases: the first one deals with the reliability analysis of the bearing capacity of the CPRF (Soil-Structure-Interaction) and the second one deals with the reliability analysis of the reinforced concrete structural elements of the CPRF.

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2. RELIABILITY ANALYSIS OF THE BEARING CAPACITY OF THE CPRF

2.1 Limit state function

On the basis of a mechanical model, the selected limit state is represented by the function:

$$G = R - S \tag{1}$$

By means of this equation, safe domains are separated from unsafe domains. The safe domain is characterized by $G > 0$ and failure occurs if $G \leq 0$.

R is the resistance of the structure and S is the action.

2.2 The resistance of the Combined Piled-Raft Foundations

The Combined Piled-Raft Foundations consist of two elements: raft and piles. Thus the resistance of the CPRF can be shown as follows:

$$R_{CPRF} = R_{raft} + R_{piles} \tag{2}$$

R_{raft} is the resistance of the raft and R_{piles} is the resistance of the piles. All variables in (**2**) are random.

Usually the resistance of raft can be considered as:

$$R_{raft} = \int_0^a \int_0^b \sigma_{raft}(x, y)\, dxdy \tag{3}$$

$\sigma_{raft}(x,y)$ - the contact pressure on the underside of the raft (random variable)
x, y - axes of the co-ordinate plane

As the first simplification the bedding modulus method (elastic bedding) is used according to the work of (Pasternak, 1925). The contact pressure can be described then as follows:

$$\sigma_{raft}(x,y) = K_S(x,y) \cdot s(x,y) = \frac{E_S(x, y)}{b \cdot f_0} \cdot s(x, y) \tag{4}$$

Where:

$K_S(x,y)$ - bedding modulus (this is the contact pressure, which causes the unit of settlement)
$s(x,y)$ - the settlement
$E_S(x,y)$ - soil stiffness (elasticity modulus)
b - Foundations width
f_0 - settlement influence value for a characteristic point according to (Grasshoff, 1955) and (Kany, 1974)

Some simplifications were made to use this equation. The settlement s is considered to be constant and deterministic for the whole footing area. The soil stiffness E_s is independent on the co-ordinates x and y but considered as random variable according to the reliability theory.

$$\sigma_{raft}(x,y) = \frac{s}{b \cdot f_0} \cdot E_S = t_0 \cdot E_S \qquad (5)$$

t_0 - a constant value and depends on the geometrical parameters

From the formulas (**4**) and (**5**) we can obtain the following equation using the formula (**3**):

$$R_{raft} = \int_0^{b_1}\int_0^{b_2} (E_S \cdot t_0)\, dxdy = E_S \cdot t_0 \cdot b_1 \cdot b_2 = E_S \cdot t_1 \qquad (6)$$

From equation (**6**) one can see that only soil stiffness E_S (as random variable) has influence on the raft bearing capacity. Therefore it is possible to present the expected value and standard deviation of the resistance of the raft with the help of the following formulas:

$$E\,[R_{raft}] = E\,[E_{S,A}] \cdot t_1 \qquad (7)$$

$$\delta\,[R_{raft}] = \delta\,[E_{S,A}] \cdot t_1 \qquad (8)$$

To take into account the spatial variability of the soil stiffness E_S it is possible to average E_S over the area A:

$$E_{S,A} = \frac{1}{A}\int_A E_S\, dA \qquad (9)$$

$E_{S,A}$ - the spatial average of E_S over the area A.

Expected value of $E_{S,A}$ will be equal to the "point expected value" $E\,[E_S]$:

$$E\,[E_{S,A}] = E\,[E_S] \qquad (10)$$

Variability of the soil stiffness E_S from point to point is measured by the standard deviation $\delta\,[E_S]$. Similarly, the standard deviation of the spatial average $[E_{S,A}]$ is $\delta\,[E_{S,A}]$. The larger the area A over which the soil stiffness is averaged, the more fluctuations of E_S tend to be cancelled in the process of spatial averaging. This tends to cause a reduction in standard deviation as the size of the averaging area increases. It is possible to present $\delta\,[E_{S,A}]$ as:

$$\delta\,[E_{S,A}] = \Gamma_{E_S}(A) \cdot \delta\,[E_S] \qquad (11)$$

where: $\Gamma_{E_S}(A)$ - reduction factor for the area A.

From the equations (**10**) and (**11**) one can get the coefficient of variation for $E_{S,A}$:

$$V\,[E_{S,A}] = \Gamma_{E_S}(A) \cdot V\,[E_S] \qquad (12)$$

For the case of the isotropic soil we get:

$$\Gamma_{E_S}(A) = \Gamma_{E_S}(b_1) \cdot \Gamma_{E_S}(b_2) \qquad (13)$$

A classical way to describe the spatial variability of E_S is using the autocorrelation function $\rho_{E_S}(\Delta x)$; Δx is the distance between the considered points. For very small intervals Δx, the coefficient $\rho_{E_S}(\Delta x)$ will be close to 1, and it usually decreases as Δx increases.

Some specific analytic expressions have been proposed for the correlation function (see for example (Vanmarke, 1977). In this work the model of (Pottharst, 1982) is used:

$$\rho_{E_S}(\Delta x) = \exp\left[-\frac{6}{x_0}\cdot(\Delta x)\right] \tag{14}$$

x_0 - extinction distance is defined as the smallest distance beyond which the correlation disappears.

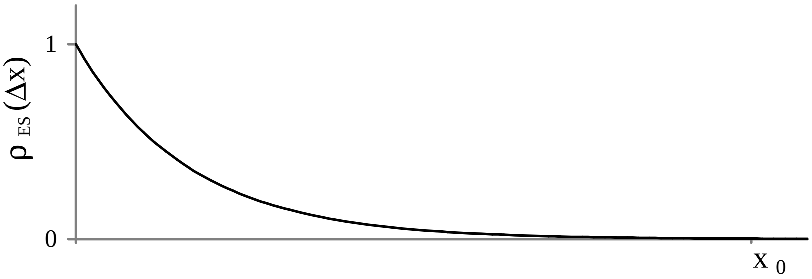

Fig. 1: Autocorrelation function and extinction distance

The type of reduction factor in equation (**13**) depends on the type of the correlation function choosed.

For a square footing with a = b the area is $A = a^2$. With the coefficient of correlation (**14**) the reduction factor is obtained as:

$$\Gamma_{E_S}(A) = \frac{x_0^2}{18\cdot a^2}\cdot\left[\frac{6\cdot a}{x_0} - 1 + \exp\left(-\frac{6\cdot a}{x_0}\right)\right] \tag{15}$$

The resistance of piles can be divided in two parts:

- Skin friction pressure and
- Foot pressure.

A common equation is the following:

$$R_{pile} = R_{skin} + R_{foot} \tag{16}$$

All variables are random.

The resistance of the skin friction pressure is:

$$R_{skin} = \int_0^{z_0} U\cdot\tau_S(z)\,dz \tag{17}$$

Where:

z_0	-	the pile depth in the soil
τ_S	-	skin friction pressure
$U = \pi\cdot D$	-	perimeter of the pile
D	-	diameter of the pile

According to the first simplification τ_S is considered to be constant over pile length but random variable. Then equation (**17**) becomes:

$$R_{skin} = U \cdot z_0 \cdot \tau_S = M \cdot \tau_S \tag{18}$$

Where: $A_S = U \cdot z_0$ - skin friction area of the pile.

With the mechanical model according to (**18**) the expected value and the standard deviation for the skin friction resistance can be written as follows:

$$E[R_{skin}] = M \cdot E[\tau_S] \tag{19}$$

$$\delta[R_{skin}] = M \cdot \delta[\tau_S] \tag{20}$$

The foot force of a pile can be presented by the foot pressure:

$$R_{foot} = \frac{\pi \cdot D^2}{4} \cdot \sigma_f \tag{21}$$

Where: σ_f - foot pressure of the pile.

Expected value and standard deviation for the pile foot force:

$$E[R_{foot}] = \frac{\pi \cdot D^2}{4} \cdot E[\sigma_f] \tag{22}$$

$$\delta[R_{foot}] = \frac{\pi \cdot D^2}{4} \cdot \delta[\sigma_f]. \tag{23}$$

2.3 Reliability analysis of Combined Piled-Raft Foundations

Taking into account the resistance of different elements of CPRF and equation (**1**) the limit state function for the bearing capacity can be written now as:

$$G = R_{raft} + R_{skin} + R_{foot} - S_G - S_Q \tag{24}$$

All variables are random and have the following probabilistic distribution functions and coefficients of variation:

- R_{raft} - resistance of the raft, lognormal distribution,
- R_{skin} - resistance of the pile from skin friction pressure, lognormal distribution,
- R_{foot} - resistance of the pile from foot pressure, lognormal distribution,
- S_G - self-weight (dead load), normal distribution,
- S_Q - live load, extreme value distribution type I,

Based on experience the ratio between the mean value of live load and the mean value of self-weight was taken as 0,20.

The limit state function (**24**) is the function of the settlements. In the design concept for CPRF both Ultimate Limit State and Serviceability Limit State are defined by the restriction of the settlements. The CPRF has to be designed on the settlement $s_{limit,\ ULS}$ for ULS and on the settlement $s_{limit,\ SLS}$ for SLS (see **Fig. 2**).

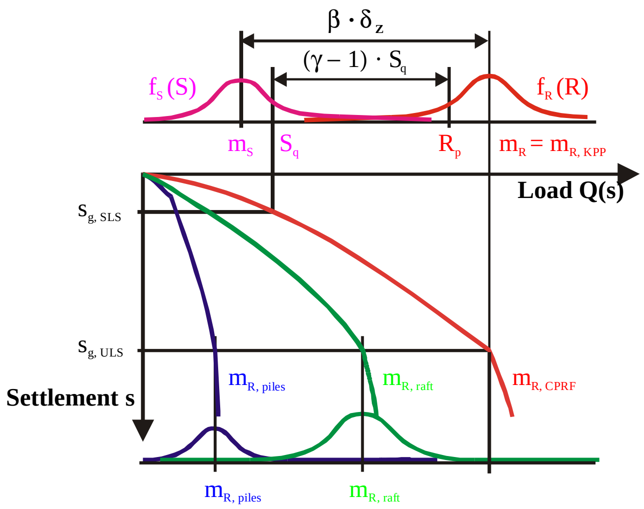

Fig. 2: Bearing capacity of a CPRF (d_Z – total standard deviation of loads and resistance)

Because of the lack of numerical values for these limiting settlements, the different levels of settlements are considered and the associated global safety factor and partial safety factors are determined.

The design values for all variables in equation (**24**) are calculated by means of the FORM (First Order Reliability Method) according to level II of the reliability theory.

As it is defined in (EC1, 1991), reliability index β is equal to 4,7 for 1 year or to 3,8 for life design working equals to 50 years.

3. PROCEDURE AND FIRST RESULTS

The CPRF is a very complex structure which consists of different components with mutual influence. Therefor, the soil-structure interaction should be taken into account. Because of difficulty of the problem it was decided to begin the investigation based on a simple model. This is an one piled-raft-foundation, which contains the basic elements of the CPRF although some phenomena which are relevant for CPRF are absent in this model.

As basis for the soil parameters the Frankfurt clay was taken since this was examined sufficiently. The calculations are represented in **table 1**. The parameters of the CPRF are presented in **fig. 3** and in the **table 2**.

The work is being done together with the Institute for Geotechnik of the University of Darmstadt. They have developed a new geophysical model for the CPRF of the basis from the Drucker-Prager model. On this basis a finite element program was written which calculats the settlements and the deformations of CPRF in Frankfurt on Main and Berlin very good.

This calculations are not based on the idealised behaviour for piles and shallow foundation as in the german code DIN but realistic behaviour was obtained by computer simulation.

soil parameter		value	
friction angle	φ´	**20**	°
cohesion	c´	**20**	kN/m2
modulus of elasticity	E	**50**	MN/m2
Poisson's ratio	μ	**0,25**	
earth pressure at rest	K_0	**0,6**	
cone-cap-factor	α	**0**	
cone angle	β	**30,64**	°
cone axis intercept	d	**32,55**	kN/m2
unit weight	γ/γ´	**19/9**	kN/m3

Table 1: soil parameter

structural element	unit	variant 1	2	3	4
pile lenth	m	15	20	25	30
pile diameter	m	0,9	1,2	1,5	1,8
slab diameter	m	6	12	-	-
slab depth	m	1	2	-	-

Table 2: investigated geometrical variants

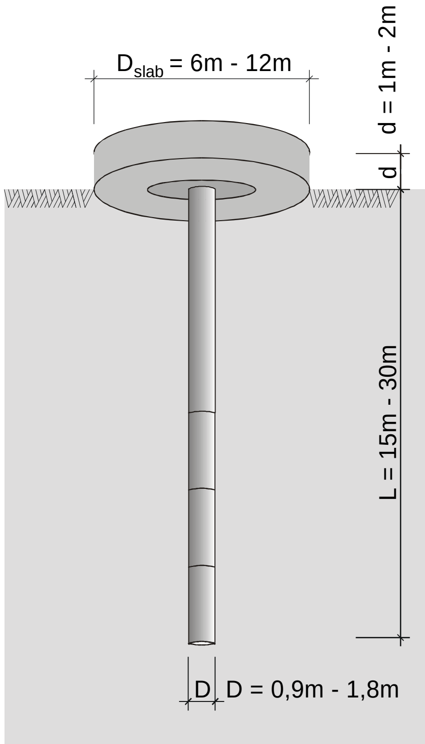

Fig. 3: system

The results for mean values of different parts of CPRF can be seen in **table 3**. It should be noted also that in the beginning of loading skin friction force is larger in comparison with the foot force. With the increase of settlements the foot pressure grows up and skin friction is reduced. For some settlements the design values of all variables and partial safety factors were calculated by means of above mentioned FORM (Level II).

Results of the calculations from Darmstadt pile length: L = 15,00 m; diameter: D = 0,90 m; raft area: A = 28,54 m²

settlement s cm	loads Q_{CPRF} MN	raft Q_{raft} MN	pile Q_{pile} MN	skin friction Q_{skin} MN	foot pressure Q_{foot} MN	loads of the components of CPRF – relation from the single loads to the loads of CPRF: raft Q_{raft}/Q_{CPRF} %	loads of CPRF: pile Q_{pile}/Q_{CPRF} %	loads of pile: skin friction Q_{skin}/Q_{pile} %	loads of pile: foot pressure Q_{foot}/Q_{pile} %
0,8	4,2	2	2,2	1,7	0,2	48%	52%	77%	9%
2	5,9	3,7	2,2	1,8	0,4	63%	37%	82%	18%
2,3	6,6	4,2	2,4	1,9	0,5	64%	36%	79%	21%
4,4	8,8	6,1	2,7	2	0,7	69%	31%	74%	26%
5	9,3	6,5	2,8	2	0,8	70%	30%	71%	29%
10	12,4	9,1	3,3	2,2	1,1	73%	27%	67%	33%
15	14,7	11,1	3,6	2,3	1,3	76%	24%	64%	36%
20	16,7	12,6	4,1	2,5	1,6	75%	25%	61%	39%
25	18	13,6	4,4	2,6	1,8	76%	24%	59%	41%

Table 3: Results of the calculation

One can see in **fig. 4** that the reduction of the coefficient of variation of the resistance parts lead to smaller partial safety factors for resistance as the same one for loads. Furthermore the investigations show that the safety factors do not change increases of the settlements.

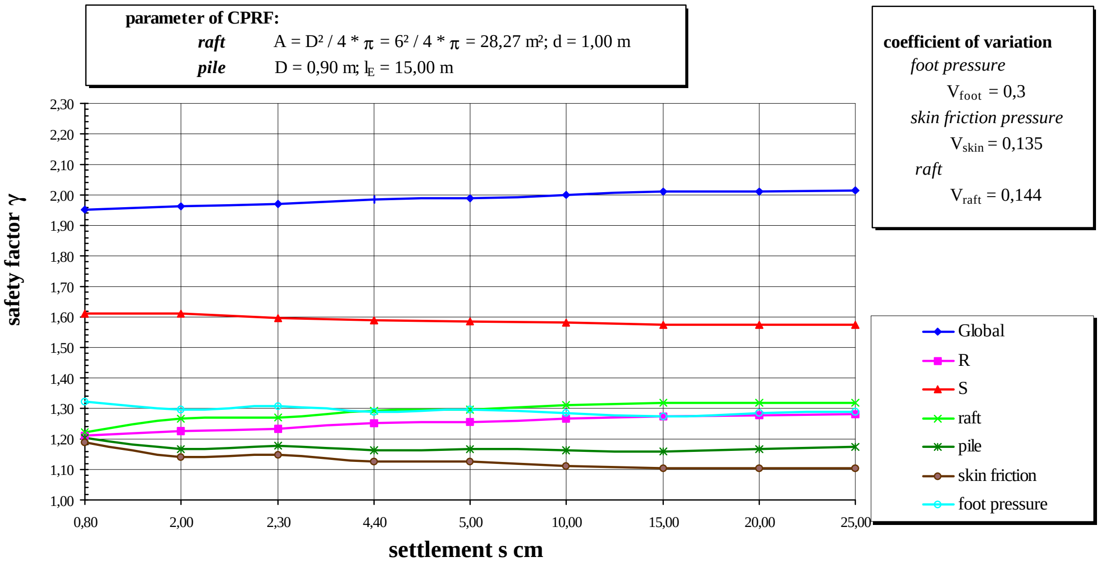

Fig. 4: Partial safety factors of the CPRF and the components

Therefore it is possible to conclude that the exact selection of the value of the settlement for the relevant limit state is not very important for the probabilistic calculations. The global safety factor obtained for all examined variant's between the values of 1,9 and 2,0. The global safety factor decreases with a lower β-value. The safety index β can be fixed for the CPRF after further investigations of the models with several piles.

4. CONCLUSIONS

The current investigations can be considered as a basis for the further work. The global safety factor for the CPRF can be assumed at first as a value of 2. It represents a good result for the transition to the single pile. The next step is the investigation of a pile -raft foundation with a plenty of piles.

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