# ANALYSIS OF THE RESULTS OF THE GPS MEASUREMENTS OBTAINED IN THE MECSEK ORE MINE COMPANY IN 1994, 1995 AND 1996 

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## SUMMARY

From 1994 till 1996 in every year were GPS measurements carried out on the movement control network in the Mecsek Ore Mine Company (Borza T., Busics I., Tóth J., Ódor K.,1996). The network consists of 5 primary sites, these sites represent the tectonical environment of the mine (Figure 1.). The deformation analysis of the results of the campaigns was accomplished in this year. The goal was to determine the deformation parameters: principal strains and their directions. In this paper I show the network, the method of the determination of the parameters, and the results.

Keywords: Global Positioning System, geodesy, deformation analysis


Figure 1: The movement control in the Mecsek Ore Mine Company

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## 1. INTRODUCTION

The GPS technology appeared in the 80 's has produced new situation and opportunities in geodesy. The GPS observations provide 3 dimensional, geocentric coordinates independently of the local gravitational forces additionally it is cheaper and faster than the classical geodetical measurements. Its accuracy overpasses that of the classical ground-observations on distances longer then 10 km .

Because of these advantages the GPS has spread in the movement controlling. This process started in Hungary, as well (Bányai L., 1993). GPS measurements were carried out on the area of the factory IV. of the Mecsek Ore Mine Company in 1994, 1995 and 1996 in order to examine the movements affected by mine operations.

## 2. THE MOVEMENT CONTROL NETWORK

### 2.1 The base and the examined points

There were established 5 examined points on the area (Kápolna, Tetõ, Völgy, Szállító). The shortest distance between the points is 412 meters longest is 1828 meters ( HP-Kápolna ). There were chosen 3 base points outside the area
û és Cserkút ). They are national-base points grounded on a mountaintop or on a high hill, $4-5 \mathrm{~km}$ far from the examined area. One of them (Cserkút) is part of the Hungarian Geodynamic Reference Network.

### 2.2 The measurements

The first measurments were carried out on $7^{\text {th }}-8^{\text {th }}$ June, 1994 so two days were enough to establish the observations. The first repeated measurments were on $25^{\text {th }}$ March, in 1995, the second was in in 1996 ( the month and the day were not available) It could be seen that the periods of the observations are much shorter than the periods between the observations. Therefore we can consider the periods of measurements points in time, in other words the displacements during the observations could be neglected. One advantage of the GPS technology is shown here as opposed to the classical measurements. It is important aspect whether the GPS tecnology can provide the right accuracy for movement controlling. The accuracy is some millimeters horizontally and larger vertically but it is still under centimeter. If the expected velocity of the movement is mm per year, then it is expedient to begin a 10 years project. In this situtation where the expected velocity is cm per year, it is enough some years to show the movements.

## 3. THEORETICAL BACKGROUND OF DEFORMATIONS

The displacement of a point can be described with $\underline{\mathbf{u}}=u \cdot \underline{i}+v \cdot \mathbf{j}+w \cdot \underline{\mathbf{k}}$ displacement vector (Kaliszky S.,1990; Nagy K., 1989) We can describe the displacement of the points of a body with the $\underline{\mathbf{u}}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\mathrm{u}(\mathrm{x}, \mathrm{y}, \mathrm{z}) \cdot \underline{\mathbf{i}}+\mathrm{v}(\mathrm{x}, \mathrm{y}, \mathrm{z}) \cdot \mathbf{j}+\mathrm{w}(\mathrm{x}, \mathrm{y}, \mathrm{z}) \cdot \underline{\mathbf{k}}$ displacement function. If we transform this function to Taylor series and considering the first order parts, the result is:

$$
u=u_{0}+\frac{\partial u}{\partial x} \cdot d x+\frac{\partial u}{\partial y} \cdot d y+\frac{\partial u}{\partial z} \cdot d z
$$

$$
\begin{align*}
& v=v_{0}+\frac{\partial v}{\partial x} \cdot d x+\frac{\partial v}{\partial y} \cdot d y+\frac{\partial v}{\partial z} \cdot d z  \tag{1}\\
& w=w_{0}+\frac{\partial w}{\partial x} \cdot d x+\frac{\partial w}{\partial y} \cdot d y+\frac{\partial w}{\partial z} \cdot d z
\end{align*}
$$

where
$\mathrm{u}_{0}, \mathrm{v}_{0}, \mathrm{w}_{0}$ - the components of the displacement of the reference point, $\mathrm{dx}, \mathrm{dy}, \mathrm{dz}$ - position-difference.
The changes of the displacements are:

$$
\begin{align*}
& d u=u-u_{0}=\frac{\partial u}{\partial x} \cdot d x+\frac{\partial u}{\partial y} \cdot d y+\frac{\partial u}{\partial z} \cdot d z \\
& d v=v-v_{0}=\frac{\partial v}{\partial x} \cdot d x+\frac{\partial v}{\partial y} \cdot d y+\frac{\partial v}{\partial z} \cdot d z  \tag{2}\\
& d w=w-w_{0}=\frac{\partial w}{\partial x} \cdot d x+\frac{\partial w}{\partial y} \cdot d y+\frac{\partial w}{\partial z} \cdot d z .
\end{align*}
$$

or in matrix-form:

$$
\begin{equation*}
\underline{\mathrm{du}}=\underline{\underline{B}} \cdot \underline{\mathrm{dr}} \tag{3}
\end{equation*}
$$

where
$\mathbf{d u}$ - changes of the displacements, $\underline{\underline{B}}$ - relative displacement tensor,
$\underline{\overline{\mathbf{d}}}$ - position-difference.
The symmetrical $\underline{\underline{\mathbf{A}}}$ and the an asymmetrical $\underline{\underline{\mathbf{R}}}$ tensors can be calculated from $\underline{\underline{\mathbf{B}}}$ :

$$
\begin{equation*}
\underline{\underline{\mathbf{A}}}=\frac{1}{2} \cdot\left(\underline{\underline{\mathbf{B}}}+\underline{\underline{B}}^{\mathrm{T}}\right) \quad \underline{\underline{\mathbf{R}}}=\frac{1}{2} \cdot\left(\underline{\underline{\mathbf{B}}}-\underline{\underline{-}}^{\mathrm{T}}\right) \tag{4}
\end{equation*}
$$

$\underline{\underline{\mathbf{A}}}$ is the deformation tensor and $\underline{\underline{\mathbf{R}}}$ is the rotation tensor and $\underline{\underline{\mathbf{B}}}=\underline{\underline{\mathbf{A}}}+\underline{\underline{\mathbf{R}}}$. The elements of $\underline{\underline{\mathbf{A}}}$ are:

$$
\underline{\underline{\mathbf{A}}=}=\left[\begin{array}{ccc}
\varepsilon_{x} & \frac{1}{2} \cdot \gamma_{y x} & \frac{1}{2} \cdot \gamma_{z x}  \tag{5}\\
\frac{1}{2} \cdot \gamma_{x y} & \varepsilon_{y} & \frac{1}{2} \cdot \gamma_{z y} \\
\frac{1}{2} \cdot \gamma_{x z} & \frac{1}{2} \cdot \gamma_{y z} & \varepsilon_{z}
\end{array}\right],
$$

where
$\varepsilon_{x}=\frac{\partial u}{\partial x}, \varepsilon_{y}=\frac{\partial v}{\partial y}, \varepsilon_{z}=\frac{\partial w}{\partial z}$ are the strains,
$\frac{1}{2} \cdot \gamma_{x y}=\frac{1}{2} \cdot\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)$ etc. are the shear between $x$ and $y$ axis.
The eigenvalues of the deformation tensor give the principal strains and the eigenvectors give their directions.

## 4. PROCESSING OF THE MEASUREMENTS' RESULTS

The obtained results are the adjusted coordinates and their mean square error for every point in every epoch in WGS -84 geocentric coordinate system. They are the startingpoint of the processing.

### 4.1 Examination of the measurements' results

The object of the study of the measurements' results is the examinations of the movement of the points. It has to be decided if the reason of the coordinate-differences between two epochs is the real displacement or the inevitable measuring error. The best

It is expedient to form the zero hypothesis that it express the immobility. During the test we compare the coordinate difference with its mean square error. Let $\Delta \mathrm{X}_{\mathrm{i}}$ the coordinate difference and $\mu_{\mathrm{Xi}}$ its mean square error. Then the zero hypothesis is:

$$
\begin{equation*}
\mathrm{H}_{0}: \quad \mathrm{w}_{\mathrm{i}}=0 \tag{6}
\end{equation*}
$$

The $w_{i}$ is a statistical function:

$$
\begin{equation*}
\mathrm{w}_{\mathrm{i}}=\frac{\Delta \mathrm{X}_{\mathrm{i}}}{\mu_{\mathrm{xi}}} \tag{7}
\end{equation*}
$$

We can make decision with the help of next criteria

$$
\left|w_{i}\right|<u_{p}
$$

where $u_{p}$ is the value of the standardized Gauss distribution on significance level p. If the zero hypothesis could be accepted, we can state that the point did not move.

In the following can be seen the most used significance levels and the corresponding criterias.

$$
\begin{array}{ll}
\mathrm{p}=0.99 & \left|\Delta \mathrm{X}_{\mathrm{i}}\right|<2.32 \cdot \mu_{\mathrm{xi}}, \\
\mathrm{p}=0.95 & \left|\Delta \mathrm{X}_{\mathrm{i}}\right|<165 \cdot \mu_{\mathrm{xi}}, \\
\mathrm{p}=0.90 & \left|\Delta \mathrm{X}_{\mathrm{i}}\right|<129 \cdot \mu_{x_{i}}, \\
\mathrm{p}=0.85 & \left|\Delta \mathrm{X}_{\mathrm{i}}\right|<104 \cdot \mu_{x_{i}} .
\end{array}
$$

During the analysis the $\mathrm{p}=0.95$ significance level was used and if the test was accepted then the coordinate difference was used further as a displacement.

### 4.2 Method of parameters' determination

During the processing the coordinate differnces were calculated in 3 combinations, then after the statistical tests, the horizontal and vertial displacements were calculated.

For example the horizontal displacements with the error ellipses between 1995 and 1996 can be seen on the Figure 2.


Figure 2: The horizontal displacements and their error ellipses between 1995 and 1996 (fixed point was Cserkút)

During the processing it was considered that the nearest points of the network forms a triangle and the deformation parameters were processed on every triangle as if it were a finite-element on a plane. This method was used because of the uniform distribution of the points.

The deformation parameters (principal strains, rotation angle) can be seen on the Figure 3. They were calculated from the horizontal displacements between the 1995 and 1996 for each triangle.


Figure 3. The deformation parameters calculated from the horizontal displacements between 1995 and 1996

The isocurves on the Figure 4 were calculated from the vertical displacements between 1994 and 1995 (in mm)


Fig. 4. The isocurves of the vertical displacements between 1994 and 1995 (mm)

## 5. CONCLUSIONS

The analysis showed that there was a more centimeters sinking on the examined area. The largest sinking is in point HP ( $\sim 5$ centimeters ) and the main feature is that the sinkings increase from point Kápolna to point HP. Horizontally the order of strains is some $\mathrm{mm} / 100 \mathrm{~m}$. The cause of the sinkings and displacements has yet to be determined by geologists and mine experts.

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