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FRACTURE MECHANICS OF BRITTLE MATERIALS: A HISTORICAL POINT OF VIEW

Laura Ceriolo[1] and Angelo Di Tommaso[2]
Venice University Institute of Architecture, Dpt. of Architectural Construction
I-30135 Venezia, Italia

SUMMARY

This paper presents milestones in the history of Fracture Mechanics (F.M.) as a tool for knowing and understanding future developments in the field. It concerns in particular lapideous materials as rocks and unreinforced concrete, which have been investigated and tested less than metals. This chapter in the science of mechanics begins with the 'energetic' approach of Alan Arnold GRIFFITH (1920) about fracture propagation and continues later on with cohesive models (A.HILLERBORG, 1976). The Griffith approach represents a new concept based on a material's discontinuity, rather than continuity, before fracture. This way allows us to understand the size effect (pointed out by R.H. LEICESTER in 1973) and snap back phenomena in brittle structural collapses.

Keywords: historical developments in F.M., brittle fracture, fracture energy, cohesive crack model, size effect, softening zone, snap back

1. INTRODUCTION

Rock blocks, used in monumental structures since the ancient time, often present unexplainable cracks. Such cracks could be provoked by work imperfections - due to technology or to installation - that lead to stress singularities. Such singularities are often the source of fracture propagation. In these lapideous materials strain localisation occurs and fracture is produced with low energy dissipation. This property characterises many huge ancient stones, selected by master builders just because easily workable.

First, primitive men "sparked" fracture energy consumed in the detachment of stone's slivers to make tools and weapons. Afterwards, stone-cutters who worked stones to make sculptures and decorations encountered an energy rate demand on the fracturing process. Such energy is now called fracture energy (GF) and it represents a material constant to characterise brittle materials.

COULOMB (1776) pioneered investigation of the fracture of stones in compression and nowadays his criterion is still used. We underline that F.M. developments are parallel to those of failure criteria for brittle materials and don't intersect each other.

Another peculiar aspect regarding brittle solids is the size effect phenomenon, interpretation of which can be done through F.M.. This was first studied by Galileo

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[1] PhD student
[2] Professor in Structural Mechanics

GALILEI, who investigated also the influence of size in fracture of structures to answer the question "why do bodies break?". Galileo, visiting the Venetian Arsenal, was surprised by the greater attention used by workers in the construction of big ships than in small ships. A master builder explained to him that it depended on greater brittleness of former compared to the latter vessels.

2. FROM GRIFFITH APPROACH TO COHESIVE CRACK MODELS

The original concept of fracture energy was conceived of by Alan Arnold GRIFFITH, a British aeronautical engineer. He was working at the Royal Aircraft Establishment (RAE) in Farnborough, where he was senior scientific officer at the Physics Department, when he investigated the fracture of glass sheets. His great contribution to ideas about breaking strength of materials was that he realised that the weakening of material by a crack could be treated as an equilibrium problem in which the reduction in strain energy of a body containing a crack, when the crack propagates, could be equated to the increase in surface energy due to the increase in surface area. The Griffith theory began from the hypothesis that brittle materials contain elliptical microcracks, which introduce high stress concentrations near their tips. He developed a relationship between crack length (a), surface energy connected with traction-free crack surfaces (2γ) and applied stress :$\sigma^2 = 2\gamma E/\pi a$

_However, the Griffith theory predicted that compressive strength of a material is 8 times greater than its tensile strength, but this condition can not be valid for any material. Later, the introduction of the line-crack by IRWIN (1957) - a flat crack which presents two singularities at the extremes - seems however be more suitable than Griffith's crack for the need to consider the friction which develops between crack surfaces. So, in 1957, George Rankine Irwin, professor of Mechanical Engineering at Lehigh University, employed also at the U.S. Navy Research Laboratory, provided the extension of Griffith theory to an arbitrary crack and proposed the criterion for a growth of this crack: the strain energy release rate (G) must be larger than the critical work (Gc), which is required to create a new unit crack area. Some say that notation G comes after Griffith, others say it is after George. Furthermore, Irwin showed, using WESTERGAARD's method, that the stress field in the area of the crack tip is completely determined by the quantity K (after KIES, a colleague of Irwin, 1952-1954), called stress intensity factor. In the parameter K subscript I refers to mode I loading, i.e. the opening mode: $K_I = \sigma \ \pi a$. Other possible modes of deformation at a crack tip are sliding mode II and tearing mode III.

Applications of F.M. to real and artificial stone materials follow with considerable delay those to metals. The rocks fracture problem takes up a prominent position regarding underground construction (mines, excavations, tunnels), to the point that in the early Sixties the first applications of Griffith's model to stone and concrete-like materials take place. Mc CLINTOCK and WALSH (1962) introduced the friction between crack faces, whereas KAPLAN (1961) focussed on the possibility of applying Linear Elastic F.M. (L.E.F.M.) to concrete. Important research about rocks were carried out by BIENIAWSKI (1967), in South Africa, where mine failures were an urgent problem to be solved.

In the same period first journals about fracture were printed (INTL. J. of F.M. in 1964, ENGINEERING F.M. in 1970) and a treatise, edited by Liebowitz, appeared in 1968.

Concerning further theoretical F.M. developments, whereby the next step was the application to concrete of parameters successfully used for metals, a new parameter for fracture was provided by J. R. RICE (1968), the so-called J-integral. This is independent of the integration path around the crack tip, also used as a crack growth criterion. It was shown that in L.E.F.M. the J-integral is equivalent to the energy release rate (G). However the J-approach doesn't give correct results for concrete or concrete-like materials because of the unloading curve characterising brittle materials and the poorly defined position of the crack tip (Hillerborg, 1983).

The debate about these problems was heated more than ever in the 80ies.

3. HILLERBORG TURNING POINT FOR BRITTLE MATERIALS

BARENBLATT (1959) and DUGDALE (1960) made the first attempt at including the cohesive forces in the crack tip region within the limits of elasticity theory. Barenblatt assumed that cohesive forces acted in a small zone (the so-called cohesive zone) near the crack ends such that the faces closed smoothly. The distribution of these forces is generally unknown. For Dugdale the distribution of the closing forces is known and constant according to an elastic-perfectly plastic material. However those models represent a limit as regards Hillerborg's model, which differs from both in several important aspects. It includes the tension softening process zone through a fictitious crack (without complete separation of its faces) ahead of the pre-existing crack whose lips are acted upon by closing forces such that there is no stress concentration at the tip of this extended crack. In the Hillerborg crack it is possible to distinguish two zones : a real crack where there is no more stresses transfer and a damaged zone, extended in the fracture process zone (F.P.Z.), in which stresses are still transferred.

The research of Arne HILLERBORG in F.M. of concrete began when he was professor in Building Materials at Lund Institute of Technology (Sweden) in the mid 70ies with the introduction of his model, suitable for concrete elements of usual size to which L.E.F.M. couldn't be applied.

"When the first RILEM technical committee on F.M. of concrete was formed in 1979 – L. ELFGREN (1991) remembers – with Professor F. H. Wittman as chairman, Hillerborg was one of the members (other prominent members were H.K. Hilsdorf, M. Lorrain, H. Mihashi, S. Mindess, A. Rosli, R.N. Swamy, S. Ziegelsdorf and A. Di Tommaso)". In 1985 he proposed a three point beam test to determine the fracture energy (GF) of concrete now accepted as RILEM recommendation. G_F represents, with the tensile strength ft and the softening law, a fracture property and it is the energy necessary to create a unit crack surface ; it is also equal to the area defined by softening law (descending branch of the (σ/w relation). In fact constitutive relation is described by a material softening law between tensile stress and local opening (width (w)of fracture process zone), instead of a stress versus strain relation for the continuous materials. This model can be applied to simulate the formation and propagation of crack using the finite element method only with inter-nodal forces. Unlike the Swedish scientist, BAZANT (1976) and Bazant and CEDOLIN (1979) used a smeared crack model to model cracking in concrete. In this model, the crack front is assumed to consist of a diffuse zone of microcracks and the stresses that close the F.P.Z. faces are represented through a stress-strain softening law. The size of this zone is related to the maximum aggregate size. An energy criterion is used for crack propagation, which can be generalized for non-linear materials behaviour. This model is particularly suitable for finite element analysis.

4. SIZE EFFECT AND SNAP BACK PHENOMENA

After Galileo, LEICESTER (1969, 1973) seems to have been the first to investigate the effect of size on the strength of structures made of metals, timber and concrete.

In order to illustrate the size dependence in a simple and dimensionless way, Hillerborg (1976) introduced the concept of a characteristic length. As a unique material property, the characteristic length $l_{ch} = G_F\ E/f_t^2$ expresses fracture of concrete and concrete-like materials, where the l_{ch} value is proportional to $f_t^{\ 2}$. This means that brittleness increases with an increase in the strength of concrete, but it decrease with a high fracture energy, according to Fictitious Crack Model (F.C.M).

The brittle response of a concrete as material can't be confused with the brittleness of a concrete structure. The brittleness of concrete structural elements depends on their size. In this way, for the same material, small elements fail with a ductile response, whereas large elements fail in a brittle manner. The variation of the structural response as the size of the structure changes is known as ductile-brittle transition. CARPINTERI (1980) proposed a parameter $s = K_{IC} / \sigma_y\ h^{1/2}$ as measure of concrete structural brittleness, but later (1986) introduced the energy Brittleness Number (or Carpinteri-number) se = GF/ft h, where h is characteristic structural dimension.

Bazant's Size Effect Law (1984) gives a measure of the brittleness of concrete elements. According to this law, $\sigma_N = Bf_t\ (1+\beta)^{-1/2}$ with $\beta = d/d_0$, σ_N = nominal strength, d= structural size, f_t = tensile strength of concrete, B and d_0 = constants, the size effect is transitional between the yield limit and the L.E.F.M. size effect.. This involves that structures with d > d0 are closer to L.E.F.M. than to yielding, so they are prevalently brittle.

The response to uniaxial tension may be unstable or catastrophic depending on size of element. This represents an instability called snap-back. This occurs when the softening branch takes on a reentrant slope in the cohesive curve with softening. Since the value of the displacement relative to the peak-stress f_t has been reached, the loading capacity falls to zero and the specimen snaps. In a uniaxial tension test displacement controlled, the stiffness falls a well as the peak stress and the dissipated energy remain constant.

5. CONCLUSIONS

Just as in structures axially compressed there is a transition from plastic collapses to instability of elastic equilibrium depending on slenderness, so in structures with prevalent tensile stress state there is a transition from plastic collapses to brittle fracture depending on the inverse of brittleness number increase.

Many catastrophic failures occurred in huge structures can be explained on the basis of F.M. concepts. Brittle materials possessing low fracture energy tend to give more dangerous brittle structural collapse, if dimension of structure is augmented.

The history of F.M. informs us that the old problem regarding brittle collapse of big structures nowadays has been clarified thanks to a scientific effort that started from Griffith's pioneering work in the 1920's and now is approaching completion.

6. REFERENCES (*)

Galilei G. (1638), "Discorsi e dimostrazioni matematiche intorno a due nuove scienze", Elzeviri, Leida (English translation, Two new sciences, The Macmillan Company, N.Y., 1933).

Coulomb C.A. (1776), "Remarque sur la rupture des corps", , Mémoires presentés par divers savants à l'Académie, vol.7.

Griffith A.A. (1920), "The phenomena of rupture and flow of solids", Phil. Trans. Roy. Soc., London.

Westergaard H.M. (1939), "Bearing pressures and cracks", J. of Applied Mechanics, vol.6.

Irwin,G.R. (1957), "Analysis of stresses and strains near the end of a crack traversing a plate", J. of Applied Mechanics, A.S.M.E., New York.

Banreblatt G.I. (1959), "Equilibrium cracks formed during brittle fracture", Prikl. Mat. Mech., 23.

Dugdale D.S. (1960), "Yielding of steel sheets containing slits", J. of The Mechanics and Physics of Solids, vol.8.

Kaplan M.F. (1961), "Crack propagation and the fracture of concrete", J. ACI, n.60.

Mc Clintock F.A., Walsh P.F.(1962), "Friction on Griffith cracks in rocks under pression", Proc. 4th National Congress Appl. Mech., Berkeley.

Bienawski Z.I., Hoek E. (1965),: "Brittle fracture propagation in rock under compression", Int. J. of F.M., 1.

Obert L. (1968), "Brittle fracture of rock", vol. VII, chap.3 in "Fracture: an advanced treatise", H. Liebowitz Editor, Academic Press, N.Y.

Bear T.J., Barr B. (1977), "Fracture toughness tests for concrete", Int. J. of F.M. vol.13.

Paul B. (1968), "Fracture: an advanced treatise", H. Liebowitz Editor, Academic Press, N.Y.

Rice J. R. (1968), "A path independent integral and the approximate analysis of strain concentration by notches and cracks", J. of Applied Mechanics, June.

Leicester R. H. (1973), "Effect of Size on the Strength of Structures", Forest Products Laboratory, Division of Building Research, Tech. Paper n°71, Commonwealth Scientific and Industrial Research Organisation, Australia.

Hillerborg A., Modeer M., Petersson P.E. (1976), "Analysis of crack formation and crack growth in concrete by means of F.M. and finite elements", Cement and concrete research, vol.6.

Carpinteri A., Di Tommaso A., Viola E.(1978), "Stato limite di frattura nei materiali fragili. Modelli meccanici teorici per l'interpretazione della frattura nei calcestruzzi e nelle rocce", Giornale del Genio Civile, apr., mag. giu, Roma.

Carpinteri A. (1980), "Notch sensitivity in fracture testing of aggregative materials", Istituto di Scienza delle Costruzioni, Università di Bologna, Nota Tecnica n°45, Jan ; Engng. F.M., vol. 16, 1982.

Carpinteri A. (1980), "Size effect in fracture toughness testing: a dimensional analysis approach ", Proc. of Intl. Conference on Analytical and Experimental F.M., Roma (Italy), Jun. 23-27.

Bazant Z.P. (1984), "Size effect in blunt fracture: concrete, rock, metal", ASCE, J. of Engng. Mechanics, vol.110.

Wittmann F.H. (1986) (editor), "Fracture toughness and fracture energy of concrete", Proc. of the Int. Conference on F.M. of Concrete, Lausanne, Switzerland, October 1-3, 1985, Elsevier Science Publishers, Amsterdam.

Di Tommaso A. (1989), Bache H., "Size effects and brittleness", in "F.M. of concrete structures : from theory to applications" (report RILEM, FMA-90), Elfgren L. (editor), Chapman and Hall, London.

Mazars J., Pijaudier Cabot. G. (1992), "Continuum damage theory : application to concrete", ASCE J. of Engng Mechanics, n 115.

Ingraffea A.R. (1990), "Case studies of simulation of fracture in concrete dams", In G-33.

Elfgren L., S.P.Shah (editors) (1991), "Analysis of concrete structures by F.M.", Chapman and Hall, London.

Carpinteri A. (1992) (editor),"Meccanica dei materiali e della frattura", Pitagora Editrice, Bologna.

Masateru Ohnami (1992) , "Fracture and Society", IOS Press, Amsterdam.

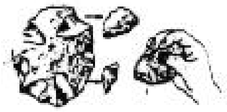
Van Mier J.G.M. (1992), "Fracture of concrete under uniaxial compression", in "Fracture processes of concrete", CRC Press Inc., Florida, USA.

Karihaloo B.L. (1995), "F.M. and Structural Concrete", Addison Wesley Longman, UK.

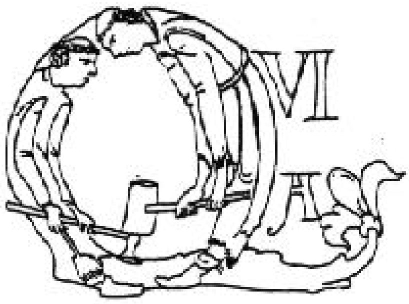
Bruwiler E. (1996), "Fracture mechanics and concrete structures: an engineering of view", in Advances in Buildings Materials Science, Gerdes A. (editor), Aedificatio Pub., D and CH.

(*) In order to illustrate the historical developments of F.M., not all references which have been selected in this list appear in the paper and are expressed in chronological order.

MILESTONES IN F.M. OF BRITTLE MATERIALS - 1



Stone Age



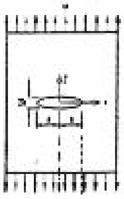
Middle Age

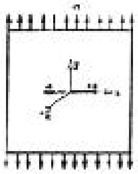
Coulomb Criterion
1776

$|\tau| = c - \sigma \cdot \mathrm{tg}\,\varphi$

Griffith Model
1920

Griffith propagation criterion
for an elliptical crack in an
elastic plate $\sigma^2 \geq 2\gamma E/\pi a$
Surface Energy $2\gamma = G$



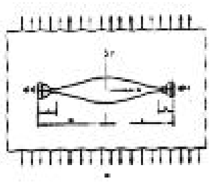


Irwin's Model
1957

Irwin line-crack

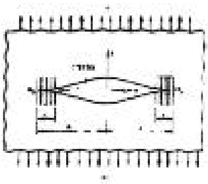
Fracture Energy $G = \dfrac{\sigma^2 \pi a}{E}$

Stress Intensity Factor $K_I = \sigma\sqrt{\pi a}$



Barenblatt Model
1959

Cohesive crack model with
unknown distibution of
cohesive forces

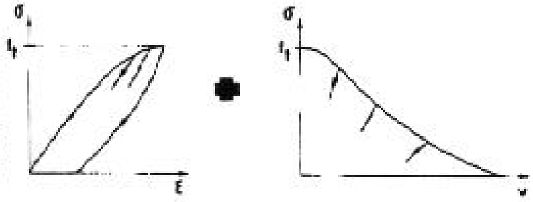


Dugdale Model
1960

Cohesive crack model with
constant distribution of closing
stresses in the fracture process
zone for elastic-perfectly
plastic materials

MILESTONES IN F.M. OF BRITTLE MATERIALS - 2

Hillerborg Model
1976



Carpinteri
Brittleness Number
1980

$$G_F = \int_0^{w_c} \sigma(w)\,dw$$

$$l_{ch} = G_F E / f_t^2$$

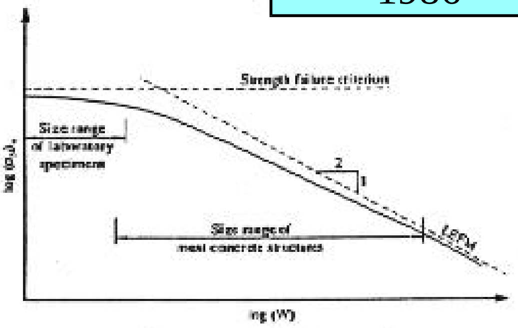
$$s = K_{IC} / \sigma_y h^{1/2}$$

Bazant Size Effect Law
1984

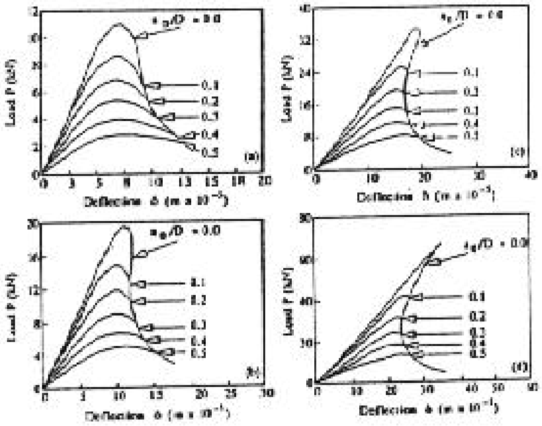
$$\sigma_N = B f_t (1 + \beta)^{-1/2}$$

Carpinteri
Energy Number
1986

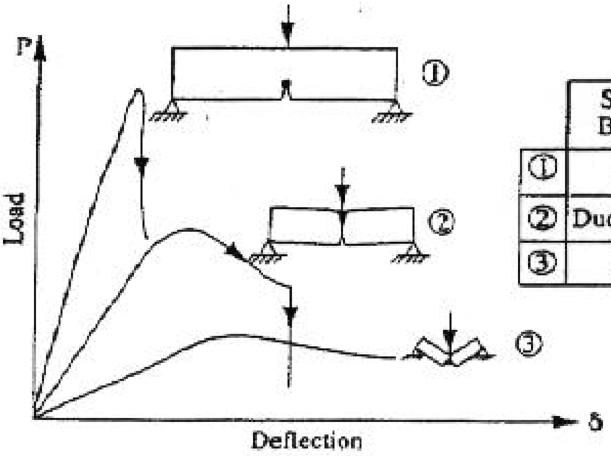
$$s_e = G_F / f_t h$$



Size Effect Law (by Bazant)



Srtuctural response of a bend plate with four different sizes (by Carpinteri)



	Structural Behaviour	Crack Growth Process
①	Brittle	Unstable
②	Ductile-Brittle	Stable-Unstable
③	Ductile	Stable

Ductile-Brittle dimensional transition in a three point bend test (by Carpinteri)