NUMERICAL MODELLING OF THE PROGRESSIVE COLLAPSE OF FRAMED STRUCTURES AS A RESULT OF IMPACT OR EXPLOSION.

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SUMMARY

A definition of progressive collapse is offered and two examples of the phenomenon described. Attention is drawn to potential sources of abnormal loads that should be examined when designing for progressive collapse performance. A plane frame computer program is described for the collapse analysis of reinforced concrete frames using finite elements and a quasi-static approach. The paper concludes by discussing the on-going development of a 3-dimensional dynamic/quasi-static non-linear finite element computer program for more accurate analysis.

Keywords: Structural Analysis, Finite Element Method, Progressive Collapse
Steel Structures, Reinforced Concrete Structures

1. INTRODUCTION

Improvements in structural analysis and knowledge of materials over the last 100 years have led engineers to build structures that are structurally more efficient than in the past. This leads increasingly to stretching constituent materials to the limit of their operational envelope. The result is that modern structures lack the strength reserve that was inherent in older structures engineered by empirical knowledge and instinct, and hence thought must be given as to how they will perform when subjected to abnormal loads. A progressive collapse occurs when a structure has its loading pattern or boundary conditions changed such that elements within the structure are loaded beyond their capacity and fail. The residual structure is forced to seek alternative load paths in order to redistribute the loads applied to it. As a result other elements may fail causing further load redistribution. The process will continue until the structure can find equilibrium either by shedding load as a by-product of elements failing or by finding stable alternative load paths.

Perhaps the most dramatic example of progressive collapse occurred in 1968 when an internal gas explosion seriously damaged the Ronan Point residential apartment block in London, UK. The explosion occurred on the 18th floor as a result of a build up of gas from a domestic cooker. The exterior panels of the kitchen area blew outwards.

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However they were designed only to withstand wind pressures applied externally, hence the continuity in the vertical load path was lost for the upper floors. The upper floor slabs failed as their outside edges were no longer supported by the exterior cladding panels. The debris from floors 18 through 22 fell on to floors 17 through to ground causing each level to suffer floor slab failure progressively as the debris load exceeded the load carrying capacity of the local structure. The result was that the entire corner of the building above and below the seat of the explosion collapsed.

Ronan Point complied with all of the prevailing local building regulations and was found to contain no notable workmanship defects. Up until that time building regulations both in the UK and USA were concerned with individual member performance and gave little consideration to the whole structure. Ronan Point was a precast large panel and slab structure with unreinforced walls and as such was stable only while continuity existed in the vertical load path. As soon as the vertical load path was disrupted and joints were expected to carry moments as the upper floors cantilevered then a disproportionate collapse was inevitable as the type of construction was analogous to a house of cards, i.e. had no demonstrable ability to redistribute loads following a local failure. Much work was done by UK code writing committees following the collapse that ultimately resulted in clauses to guard against disproportionate collapse being adopted in to the 1975 UK Building Regulations. The clauses cover horizontal and vertical continuity, horizontal loading, ductility. For structures greater than 5 stories, where ties do not reach the desired minimum, any single vertical structural member must be able to be rendered incapable without causing significant collapse. Where any vertical element may not be removed, it and its connections must be able to withstand a specified overpressure applied in any direction, Popoff(1975).

The abnormal loading of structures is not confined to high-rise buildings. The Piper Alpha oil rig disaster in the British North Sea during the 1980s highlighted the extreme vulnerability of offshore structures to catastrophic loading events. Terrorist activity has also underlined the need for structures to behave in a predictable manner in order to optimise the survival chances of the occupants. The performance of the federal government building following the Easter 1995 fertiliser truck bomb in Oklahoma City, USA, tragically demonstrated the lack of thought given to structural behaviour outside of the usual design loads. The federal building had won awards for its design and by any standards was a visually appealing use of precast concrete, but the vulnerability was also obvious. Columns from the upper floors terminated at a spandrel beam on the first floor below which the columns were at wider centres. The truck bomb shattered the spandrel beam and as a result support to the upper floor columns was lost and the entire elevation failed, and as a result of the loss of the exterior elevation the floor units and secondary beams failed to a distance of one bay in to the building interior.

It is now clear that abnormal loadings must be taken into account when designing structures. Abnormal load events come from a number of sources: gas explosion; confined dust or vapour conflagration; machine malfunction; bombs; or projectile impact. Given the sensitivity of modern business to disruption either of staff or information then knowledge of how a given structure will perform under a particular set of conditions is of prime importance when calculating company losses due to unforeseen occurrences. It is therefore desirable that engineers should have at their disposal tools that enable them to analyse their structures cost-effectively with regard to performance under abnormal loading conditions without demanding full familiarity with the complexities of the problem. To date no single tool exists that can perform a progressive collapse analysis. To that end the Structures Research Centre at City University has developed a program for the analysis of plane reinforced concrete frames and is currently engaged in developing a computer program capable of analysing both steel and concrete 3-dimensional frames with regard to progressive collapse performance.

2. PLANE FRAME ANALYSIS

The reinforced concrete plane frame solution developed at City University by Virdi and Beshara (1992) follows a quasi-static approach in analysing structural behaviour. The benefits of a quasi-static approach as opposed to a full dynamic analysis are twofold. Firstly, the problem formulation is the same as for a regular static analysis, and secondly the lack of dynamic terms in the equations means that the computer time required to obtain a solution is very much reduced. The drawbacks of using such an approach in areas of a structure exposed to true dynamic loading, i.e. those members local to the explosion or impact, have been highlighted by Jones (1995) and Pretlove *et al* (1991). Both workers point out that the quasi-static approach is non-conservative because it ignores the effects of energy released in the system as members fail. The energy can cause transient loads and displacements greater than those apparent from the static case, and hence the quasi-static approach may miss some potential member failures or elect to remove members in the wrong order. Jones gave an example of an impact on a beam. Dynamically the response would be governed by the beam's response to transverse shear, with transverse shear failure a possibility. Whereas in static analysis transverse shear effects would not be a critical parameter.

The computer program *PROCSIE* developed at City University by Jeyarupalingam (1993) and Gilmour (1998) takes a quasi-static non-linear finite element approach to modelling structural behaviour. The three main facets of the program are:

- local damage analysis
- alternative load path analysis
- debris load analysis.

Local damage assessment is carried out using the single degree of freedom, SDOF, approach suggested by Biggs (1964). Other more rigorous techniques were examined including Dyna-3D and FEABRS (a program developed by Beshara (1991) for blast analysis of RC beams). A parametric study carried out by Iyengunmwena (1991) showed that Biggs' method was accurate to within 8% in most cases. A member is selected by the user to be subject to the abnormal load. It is then analysed by Biggs' method and end moments and shears are determined. At the instant immediately prior to failure the frame will be reacting equal and opposite shears and moments on the member.

In order to obtain the quasi-static response of the rest of the structure, the frame is analysed with the member removed and the end moments and shears applied initially at 50% of their magnitude. Assuming equilibrium can be found then the abnormal load factor is incremented up until it reaches 100%. At each iteration the program monitors the strain at specific cross-sections located in all members checking for additional member failures as a result of the applied abnormal end reactions and redistributed loads.

Alternative load path analysis refers to the removal of elements from the frame that have failed the strain limit criteria at the cross-sections along the member. The failure criteria used was that strains greater than concrete crushing strain when detected at both ends of a member implied failure of the complete member. Such a member is then removed at the next iteration. The frame is re-analysed and loads are redistributed by the user to the surrounding frame elements as appropriate. This continues until the abnormal load factor reaches unity and no further members breach the strain failure criteria.

Debris load analysis is carried out by the user when a vertical load carrying member is removed, e.g. a beam fails and it is necessary to know where to put the beam's imposed and dead load. The debris loads are factored up by 1.25 in order a take in to account the effects of impact from the falling debris. The load vector entry is updated for the member to which the debris has been directed, and the analysis continues until a steady state is reached.

A number of assumptions have been made in order to allow for plane frame modelling to take place:

- The cross-section is subjected to uniaxial or biaxial bending moment and axial force.
- shear stresses are neglected
- plane sections remain plane before and after bending
- the frame is made up of beam-column finite elements
- each member is considered to be shallow with respect to its length, and so can be represented entirely by its mid-height longitudinal axis.
- out of plane displacements are neglected.
- loads are applied only in the plane of the frame.

The finite element scheme adopted is relatively straight forward. The frame members are modelled by evaluating the displacement field at a number of stations along the member length using Gauss integration techniques. At each of the Gauss points along the member the entire cross-section is evaluated and a biaxial grid of Gauss points is used to integrate over the section to obtain the required stress resultants. The value of Young's modulus at each Gauss point on the cross section is monitored and from an assumed strain displacement relationship the stress at each Gauss point can be evaluated using the selected material relationship.

The direct method is employed for the resolution of the equation of equilibrium;

$$\left[K_s\right]\left\{Q\right\} = \left\{F\right\} \qquad [1]$$

K_s is the global secant stiffness matrix, and is determined by iteration as initially the internal forces and displacements are unknown. The following has been employed as a solution procedure:

1. form the elastic stiffness matrix for each element assuming no axial force and elastic material response.
2. form the transformation matrix from the initial undeformed geometry and form the global stiffness matrix.
3. assemble the applied load vector F
4. solve the equations for the unknown degrees of freedom,

$$[K_s]_{i-1}\{Q\}_i = \{F\} \qquad [2]$$

   note that K lags behind the current iteration i.
5. using Q_i obtain updated material state, strains, stresses and rigidity matrices.
6. re-form: element stiffness matrices, transformation matrices, and global stiffness matrix K_s.
7. using Q_i and K_s calculate the member forces at each node

$$\{F\}_i = [K_s]_i\{Q\}_i \qquad [3]$$

8. compare applied load vector F and calculated F_i, continue to iterate from step 4 to step 7until the external and internal forces converge.

A graphical interface was written for use with the Fortran program using Visual Basic. The interface allows for easy generation of data files, graphical representation of the data, and plotting of the deflected shape, bending moment, shear force and axial force diagrams.

3. THREE-DIMENSIONAL ANALYSIS

In order to increase the accuracy and counter the problem of using quasi-static analysis over truly dynamic regions a 3-D dynamic/quasi-static non-linear finite element code is being developed. It is proposed to decompose the whole structure in to two regions, a dynamic region and a quasi-static region. The dynamic region will consist of those members subject to the abnormal dynamic load and members immediately connected to them. Ideally the boundary between dynamic and quasi-static regions would be decided by an initially large region dynamic analysis, that is then subsequently reduced in size depending on the strain rates reported by the analysis. In the short term the decomposition will be done by user judgement. The dynamic analysis will be carried out by Dyna-3D, a well known explicit FE code developed for weapons research in the 1980s. The code has the ability to stop prior to each time-step and receive updated boundary conditions, e.g. force and displacement data for the nodes common to the dynamic and quasi-static regions. The progressive collapse analysis begins with a dynamic analysis of the selected region. Once equilibrium is obtained for a given time-step the quasi-static region is analysed until it is in equilibrium. The quasi-static analysis uses the forces and displacements of the nodes common to both regions as prescribed force and displacement values to begin the analysis. Any revision to these nodal values is passed back to the dynamic region when updating the boundary conditions for the next time-step. Both types of analysis monitor their constituent members for compliance with failure criteria, which if broken results in the member concerned being removed and its loads redistributed.

The 3-dimensional quasi-static formulation currently under development allows for the inclusion of a number of non-linear effects and interactions that are not possible to model in 2-dimensional analysis, e.g.

- analysis in true 3-dimensional space
- warping of sections
- lateral-torsional buckling
- out of plane loading.

In order to encompass these effects a number of high order terms were retained in the general strain equation. (symbols defined in Section 5)

$$
\begin{aligned}
e_z = {} & u_0' - yv_0'' - xw_0'' + wq_z'' + \frac{1}{2}\left(u_0'\right)^2 - yu_0'v_0'' - xu_0'w_0'' + \frac{1}{2}\left(yv_0''\right)^2 + \frac{1}{2}\left(xw_0''\right)^2 + \frac{1}{2}\left(v_0'\right)^2 \\
& - \frac{yv_0''\left(v_0'\right)^2}{\sqrt{1-\left(v_0'\right)^2}} + v_0'xq_z'\sqrt{1-\left(w_0'\right)^2} + \frac{1}{2}y^2\frac{\left(v_0'\right)^2\left(v_0''\right)^2}{1-\left(v_0'\right)^2} + \frac{1}{2}x^2\left(q_z'\right)^2\left(1-\left(w_0'\right)^2\right) + \frac{1}{2}\left(w_0'\right)^2 \\
& - \frac{x\left(w_0'\right)^2 w_0''}{\sqrt{1-\left(w_0'\right)^2}} - w_0'yq_z'\sqrt{1-\left(v_0'\right)^2} + \frac{1}{2}x^2\frac{\left(w_0'\right)^2\left(w_0''\right)^2}{1-\left(w_0'\right)^2} + \frac{1}{2}y^2\left(q_z'\right)^2\left(1-\left(v_0'\right)^2\right) + u_0'q_z''w
\end{aligned}
$$

[4]

The strain equation [4] is based on the geometric derivation of Saada (1993) and on the warping hypothesis of Vlasov (1961), the equation has been successfully applied to large strain large displacement analysis by Bailey *et al* (1996). If the shape functions describe the displacement of an arbitrary reference axis then the strain at any point in the I-section may be determined by inserting the relevant (x,y) coordinates of the point with respect to the reference axis and the sectorial coordinate ω of that point. The equation includes the terms necessary to represent thin-walled open sections and so certain terms may be omitted when evaluating torsionally stiff rectangular beams where warping is not significant. The principle of virtual work is applied over the length of the beam-column elements, equation [5], from which point the solution follows the usual pattern as described by many workers, e.g. Zienkiewicz (1991).

$$dW = \int_V s_z de_z + t_{xz} dg_{xz} + t_{yz} dg_{yz} dv - \langle Q \rangle \{dq\} = 0 \qquad [5]$$

The finite element solution of these equations is much the same as that applied in the plane frame case. Standard cubic shape functions are used to determine the displacement field and Gauss integration is employed to integrate along the member length. At each of the longitudinal Gauss points the reference axis displacement is used to evaluate the strain across the cross-section according to a biaxial grid of Gauss points describing the section. Integration is carried out over the cross-section in order to determine the stress resultants required to form the stiffness matrix. The descritisation of the cross-section in this manner allows for the spread of plasticity to be dealt with as each Gauss point may have a different modulus of elasticity according to the user selected material relationship, this also allows for the use of composite sections. In order to monitor the behaviour of the beam-column near joints the individual frame members are modelled by 3 beam-column elements as suggested by El-Zanaty and Murray (1983). The 3 elements are arranged such that their total length is equal to the member length, but their individual lengths vary. The member is sub-divided into 10% + 80% + 10% of the total length, each percentage length being modelled by one element. In order not to increase the total number of unknowns the internal degrees of freedom are condensed out. Since point loads must be applied at nodes then the method guarantees a large number of Gauss points in the member lengths either side of a point load, thus hinge formation and spread of plasticity can be traced. This is important because the abnormal loads used in the analysis will tend to cause extensive plastification of sections as material failure begins to occur.

The key to modelling the progressive collapse phenomena correctly is the application of failure criteria. The term *failure* is itself rather misleading as it is generally applied to a material when the material stops acting in the manner in which it was expected to act at design. In actual fact the material or rather the member composed of that material may still be capable of influencing the distribution of forces in its locality. This is particularly the case for reinforced concrete sections that have begun to crack and have lost the majority of their tensile and flexural load carrying capacity but can still bear compressive forces and some tensile force until ductile failure of the reinforcement occurs. The point at which steel members are removed is a little easier to deal with:

- comparing the stresses in the section with one of the established yield models
- employing limiting rotations at weld sites.

One must also remember that concrete is often used in high risk structures due to its ability to absorb energy during fracture. Therefore some energy wasting function should be incorporated in to the concrete model. The form that such a function should take is as yet undecided although its justification follows along the lines of the smeared crack approach, i.e. specific crack paths are not important but accounting for crack energy is. The smeared crack approach cannot be employed 'as is' since insufficient strain data is available along the length of the member as a result of the one-dimensional elements used to represent the beam-columns. The use of such elements is necessary in order to allow for whole structure analysis within a reasonable time on a PC or single processor workstation.

4. CONCLUSION

The plane frame analysis described now works. Although not fully automatic, the analysis gives a good graphic visualisation of the phenomenon of progressive collapse. The 3-D analysis under development will integrate the local dynamic and the global quasi-static analysis. The formulation retains a number of higher order terms in order to model structural behaviour more accurately when large strains are present. Consideration of member behaviour has been improved, and will allow for warping of thin-walled open sections and lateral-torsional buckling of both thin-walled open sections and rectangular reinforced concrete sections. The proposed improvements to be incorporated in to the 3-D analysis will offer a more realistic evaluation of the progressive collapse phenomenon.

5. NOTATIONS

u_0, v_0, w_0	displacement of reference axis (z,y,x)
ω	sectorial coordinate
x, y	cartesian coordinates of a point on the cross-section with respect to the reference axis
θ_z	rotation of the cross-section along the z-axis
ε_z	axial strain
σ_z	axial stress
$\langle Q \rangle$	row vector of external forces
$\{\delta q\}$	column vector of incremental deformations
δW	increment of work
$\delta\varepsilon_z$	incremental variation in axial strain
τ_{xz}, τ_{yz}	shear stress
$\delta\gamma_{xz}$, $\delta\gamma_{yz}$	incremental variation in shear strain
$'$	first derivative with respect to ∂z
$''$	second derivative with respect to ∂z

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