SYSTEM IDENTIFICATION OF DAMAGE IN REINFORCED CONCRETE STRUCTURES BY MEASURED MODAL TEST DATA

Thomas Jahn[1] and Gerhard Mehlhorn[2]
University of Kassel, Department of Structural Concrete,
Kurt-Wolters-Str. 3, D-34109 Kassel, Germany.

SUMMARY

Structural damage in reinforced concrete structures usually causes a local reduction in stiffness. Often the defects are not visible on the surface of the structures. Experimental dynamic measurement makes it possible to judge the structure state. The degree of damage can be given by the definition of damage parameters. This article describes a procedure, to identify the parameters using measured modal test data (eigenfrequencies and eigenvectors). The estimated starting parameters will be corrected by minimisation of differences between measured and corresponding analytical data.

Keywords: structural damage, reinforced concrete, crack

1. INTRODUCTION

The objective of this work is to show that the measurement and identification of eigenfrequencies and eigenvectors can be used to draw conclusions about damage in reinforced concrete structures. Such damage can be, for example, a partial or complete failure of reinforcement. The causes of these defects are errors in construction planning, material defects or corrosion. The corrosion of steel is directly connected with cracks in the concrete.

As a result of loads, temperature or shrinkage cracks reach such a size that carbon dioxide, water and oxygen penetrate to the reinforcement. The area of protective alkaline milieu will be destroyed, and the reinforcement begins to corrode.

The formation of cracks in reinforced concrete is unavoidable, so it is necessary to judge and limit the cracks' size with respect to the structure's appearance, the corrosion of the reinforcement and the permeability. As a rule, narrow cracks (≤ 0.3 mm) have no influence on the corrosion of the reinforcement. The term „damaging crack“ can only be used when the cracks are so large that they affect the usefulness or load capacity of the structure.

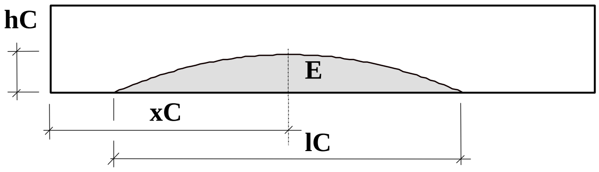
Cracks cause a local reduction in stiffness. This stiffness reduction changes the dynamic behaviour of the structure. This paper describes a procedure which solves the inverse problem to this observation: the use of dynamic test data (eigenfrequencies and eigenvectors) to judge the extent of crack damage. This procedure belongs to the group of indirect parameter correction methods.

---

[1] Dr.-Ing.,
[2] Prof. Dr.-Ing. Dr.-Ing. E.h., Professor in Structural Engineering.

2. IDENTIFICATION OF PARAMETER OF THE CRACK AREA

The area of damage is an area in which the assumed homogeneous structure has a defect, caused by changing material or geometrical properties.
The starting point of the test is a reinforced concrete beam in a cracked state. The cracks are the defects of the assumedly homogeneous macrostructure.
Under the assumption that the crack area is dominated by bending cracks, it is approximated by a quadratic parabola. Then the area can be described by geometrical and material parameters.
The beam structure will be mapped into a finite element model, where the damage is modelled by the following parameters (Fig.(1)):



hC ... height of the crack area,
lC ... length of the crack area,
xC ... position of the crack area and
E ... modulus of elasticity.

Fig.1: Parameters of the crack area

3. MATEMATICAL BASIS FOR THE CORRECTION OF PARAMETERS

The vector of the residuals R_W serves as the basis for correcting the parameters. In R_W the weighted errors from measured and calculated parameters are grouped together [2]:

$$\mathbf{R}_w = \hat{\mathbf{W}} \cdot \mathbf{R} = \hat{\mathbf{W}} \cdot (\mathbf{V} - \mathbf{V}^C(\mathbf{P})), \qquad (1)$$

(**R** ... vector of residuals, **V** ... vector of experimentally measured values, $\mathbf{V}^C(\mathbf{P})$ ... vector of corresponding calculated values, **P** ... vector of the damage parameters, $\hat{\mathbf{W}}$ ... weighting matrix).

The parameters are estimated by the least square method:

$$\mathbf{J} = \mathbf{R}_w^T \cdot \mathbf{R}_w \qquad \rightarrow \qquad \text{Min}. \qquad (2)$$

A minimization of J with respect to the residuals provides equations for calculating parameters. $V^C(P)$ in Eq. (1) is a nonlinear function of the damage parameters P. It is linearised by a Taylor series truncated after the linear term according to:

$$\mathbf{V}^C(\mathbf{P}) = \mathbf{V}_a + \mathbf{G} \cdot \Delta\mathbf{P}, \qquad (3)$$

where $\mathbf{V}_a = \mathbf{V}^C\big|_{\mathbf{P}=\mathbf{P}_a}$ and $\mathbf{G} = \left.\dfrac{\partial \mathbf{V}^C}{\partial \mathbf{P}}\right|_{\mathbf{P}=\mathbf{P}_a}$,

(G ... sensitivity matrix, $\Delta\mathbf{P}$ ... parameter changes (=$\mathbf{P}$-$\mathbf{P}_a$), $\mathbf{P}_a$ ... linearisation point).

Then is: $\mathbf{R}_W = \hat{\mathbf{W}} \cdot (\mathbf{V} - (\mathbf{V}_a + \mathbf{G} \cdot \Delta\mathbf{P}))$. *(4)*

The necessary conditions for minimizing Eq. (2) is:

$$\frac{\partial \mathbf{J}}{\partial \mathbf{P}} = \frac{\partial \mathbf{R}_w^T}{\partial \mathbf{P}} \cdot \mathbf{R}_w + \mathbf{R}_w^T \cdot \frac{\partial \mathbf{R}_w}{\partial \mathbf{P}} = 2 \cdot \frac{\partial \mathbf{R}_w^T}{\partial \mathbf{P}} \cdot \mathbf{R}_w = \mathbf{0}. \qquad (5)$$

Substitution of Eq. (1) into Eq. (5) yields:

$$-2\cdot\frac{\partial \mathbf{V}^{K}(\mathbf{P})^{T}}{\partial \mathbf{P}}\cdot\mathbf{W}\cdot(\mathbf{V}-\mathbf{V}^{K}(\mathbf{P}))=\mathbf{0} \qquad (6)$$

where $\mathbf{W}=\hat{\mathbf{W}}^{T}\cdot\hat{\mathbf{W}}$.

The parameter variances $\Delta\mathbf{P}$ are obtained from the Eq. (3) and (4):

$$\Delta\mathbf{P}=(\mathbf{G}^{T}\cdot\mathbf{W}\cdot\mathbf{G})^{-1}\cdot\mathbf{G}^{T}\cdot\mathbf{W}\cdot(\mathbf{V}-\mathbf{V}_{a})\,. \qquad (7)$$

This equation is solved iteratively:

$$\mathbf{P}_{i+1}=\mathbf{P}_{i}+\Delta\mathbf{P}_{i}\,. \qquad (8)$$

Using the differences between measured and calculated eigenvalues (λ, λ^{C}) and eigenvectors ($\mathbf{X}$, $\mathbf{X}^{C}$), the residual vector in Eq. (1) is:

$$\mathbf{R}=(\lambda_{1}-\lambda_{1}^{C},\ldots,\lambda_{n}-\lambda_{n}^{C},\mathbf{X}_{1}-\mathbf{X}_{1}^{C},\ldots,\mathbf{X}_{n}-\mathbf{X}_{n}^{C})\,. \qquad (9)$$

The solution of the eigenvalue problem of the corrected analytical model produces the dynamic parameters (λ^{C}, $\mathbf{X}^{C}$):

$$(\mathbf{K}^{C}-\lambda_{i}^{C}\cdot\mathbf{M}^{C})\cdot\mathbf{X}_{i}^{C}=\mathbf{0}\,, \qquad (10)$$

(λ_{i}^{C} ... corrected eigenvalue, i=1,...,n, $\mathbf{X}_{i}^{C}$ ... corrected eigenvector, $\mathbf{M}^{C}$ ... corrected mass matrix,$\mathbf{K}^{C}$ ... corrected stiffness matrix).

Assuming that the mass matrix is constant ($M=M^{C}$) and independent of the parameter changes, then only a correction of the stiffness matrix is necessary. Corresponding to the Eq. (6) the partial derivatives of the residual vector with respect to the parameters P are:

$$\frac{\partial\mathbf{R}}{\partial\mathbf{P}}=\left(\frac{\partial\lambda_{1}^{C}}{\partial\mathbf{P}},\ldots,\frac{\partial\lambda_{n}^{C}}{\partial\mathbf{P}},\frac{\partial\mathbf{X}_{i}^{C}}{\partial\mathbf{P}},\ldots,\frac{\partial\mathbf{X}_{n}^{C}}{\partial\mathbf{P}}\right). \qquad (11)$$

The partial derivatives of the eigenvalues λ^{C} and of the eigenvectors $\mathbf{X}^{C}$ (compare [7]) are:

$$\frac{\partial\lambda_{i}^{C}}{\partial\mathbf{P}}=\mathbf{X}_{i}^{CT}\cdot\frac{\partial\mathbf{K}^{C}}{\partial\mathbf{P}}\cdot\mathbf{X}_{i}^{C}\,,\qquad\qquad \frac{\partial\mathbf{X}_{i}^{C}}{\partial\mathbf{P}}=\sum_{\substack{i=1\\ i\neq j}}^{n}\frac{\mathbf{X}_{i}^{CT}\cdot\frac{\partial\mathbf{K}^{C}}{\partial\mathbf{P}}\cdot\mathbf{X}_{j}^{C}}{\lambda_{j}^{C}-\lambda_{i}^{C}}\mathbf{X}_{i}^{C}\,. \qquad (12),(13)$$

4. FINITE ELEMENT MODEL

The theoretical basis for the FE-model is described in the literature [1]. its important properties will be described here.

An isoparametric plane stress finite element (4-node) is used to model the concrete structure and smeared crack model to represent the cracks in the beam [8]. The reinforcement is modelled by 2-node-bar elements. The bond between concrete and steel is assumed to be perfect (no relative displacements between concrete and steel). The cracked reinforced concrete is modelled using two material properties. As the forces and displacements are very small, it is assumed that the material is linear elastic.

In the areas where the beam is uncracked, isotropic material behaviour is assumed, Eq. (14). In cracked areas, orthotropic material behaviour is assumed, Eq. (15).

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}, \qquad \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix}E_{11} & E_{12} & 0\\ E_{21} & E_{22} & 0\\ 0 & 0 & E_{33}\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}, \qquad (14),(15)$$

(σ_{xx}, ε_{xx} ... normal stresses and strains in x-direction, σ_{yy}, ε_{yy} ... normal stresses and strains in y-direction, σ_{xy}, ε_{xy} ... shear stresses and strains in x- and y-direction).

For steel, linear elastic material behaviour is assumed:

$$\sigma_{xx} = E_s \cdot \varepsilon_{xx}\,, \qquad (16)$$

(E_s ... elasticity modulus of steel, σ_{xx}, ε_{xx} ... stresses and strains in x-direction).

5. EXAMPLE OF THE PARAMETER CORRECTION

The correction of parameters of a crack area is described by a mathematical algorithm [4] in the programming language [6].

The example demonstrates the parameter correction of a crack area for a real reinforced concrete test beam. The crack area has been produced by a point load applied at the third-points of the beam.

The dynamic behaviour of the undamped beam has been identified in the uncracked and cracked state using the computer program [3], [5]. In the dynamic tests both sides of the beam have been freely supported. The eigenfrequencies of the uncracked structure have been used to determine the average dynamic elastic modulus of the concrete beam. This modulus (E=35.5 N/mm^2) has been used for the uncracked area of the beam during the identification of the parameters. The parameters of the crack area have been corrected by using the first five modes of the test model. Table 1 gives the estimated start and the corrected parameters (29th iteration step).

Table 1

Parameter of crack area	**start values**	**corrected values**
length of the crack area lC [m]	1.00	1.54
height of the crack area hC [m]	0.12	0.12
position of the crack area xC [m]	1.00	1.74
E-Modulus E [kN/mm²] (vertical to the crack direction)	30.0	17.1

The differences of the eigenfrequencies of the test and of the analytical models prior to the 1st step and following the last (29th) step are shown in Table 2.

The so-called MAC-value (Modal Assurance Criteria) in Eq. (17) [2] allows a judgement of the comparability of the modal data:

$$MAC = \frac{(\mathbf{X}^{CT}\cdot\mathbf{X})^2}{(\mathbf{X}^{CT}\cdot\mathbf{X}^C)\cdot(\mathbf{X}^T\cdot\mathbf{X})}\cdot 100[\%], \qquad (17)$$

($\mathbf{X}^C$ ... eigenvector of the analytical model, $\mathbf{X}$ ... eigenvector of the test model).

The value of the MAC is between 0 and 100%. A value of 100% means that the modes correspond exactly.

A necessary condition for the identified parameters is that all modal responses of the test model agree with the results of the analytical model. The shaded modal responses of the test model (Table 2) have not been used to correct the parameters. These values agree very well with the calculated ones. This comparison evaluates the quality of the identified parameters.

Fig. 2 shows the parameter variations with respect to the start values, the differences of frequencies and the MAC-values in every iteration step.
Fig. 3 represents the cracks in the tested beam and the adapted quadratic parabola (drawn according to scale) as a result of the parameter correction process.

Table 2: Differences between eigenfequencies and MAC-values

Iteration step	Number of Mode	Eigenfrequencies [Hz] Test model	Eigenfrequencies [Hz] corr. model	Differences [%]	MAC-values [%]
0	1	88.34	100.9	+14.22	99.46
	2	245.3	272.3	+11.01	99.02
	3	465.6	516.6	+10.95	98.47
	4	767.3	820.7	+6.96	97.53
	5	1085	1166	+7.47	97.98
	6	1434	1548	+7.95	97.17
	7	1813	1955	+7.83	95.65
29	1	88.34	88.46	+0.14	99.94
	2	245.3	249.4	+1.67	99.64
	3	465.6	473.3	+1.65	99.40
	4	767.3	755.9	-1.49	99.05
	5	1085	1090	+0.46	99.39
	6	1434	1452	+1.26	99.48
	7	1813	1845	+1.77	99.15

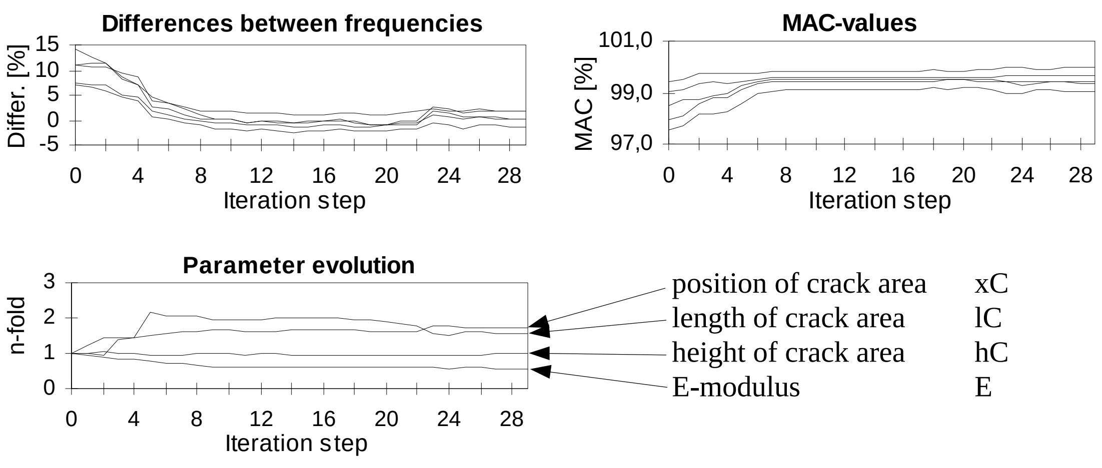


Fig. 2: Parameter correction

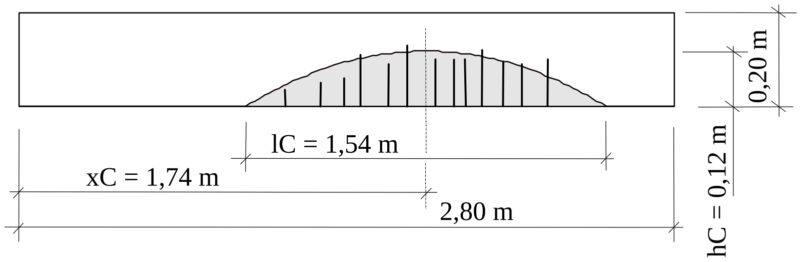


Fig. 3: *Crack pattern and identified crack area*

6. CONCLUSIONS AND RECOMMENDATIONS

The example of the parameter correction process demonstrates that it is possible to approximate the localisation and determine the crack area dimensions with a quadratic parabola. A disadvantage of this method is that the crack area will always be a parabolic shape, even if the crack area has another form. The results of this correction model do not necessarily describe the real shape of the area.
A recommended future research plan is to conduct experiments, carried out so far under laboratory conditions of individual structure elements, also of more complete structures. Based on the identified damage it is possible to judge the load capacity and the life expectancy of the structure. In this research the following unsolved problems still have to be examined:

- structures with multiple crack areas and other support conditions,
- the possibility to identify cracks produced by shear loads with the modal analysis methods and
- the estimation of the reliability of the identified results by means of statistical analysis.

7. REFERENCES

1. Bathe, K.-J., Finite Element Procedures in Engineering Analysis, *Prentice Hall*, 1982.
2. Friswell, M.J., Mottershead, J.E., Finite Element Model Updating in Structural Dynamics, *Kluwer Academic Publishers*, Dordrecht 1995.
3. ISSPA, Program system for the identification of modal parameters, *Program-documentation*, FG Leichtbau, Universität Gesamthochschule Kassel.
4. Jahn, T., Ein Beitrag zur Identifikation von Schädigungsparametern an Stahlbetonbauteilen, *Dissertation*, Universität Gesamthochschule Kassel 1996.
5. Link, M., Qian, G., Identification of dynamic models using base excitation and measured reaction forces, *Revue Française de Mécanique*, n° 1994-1.
6. MATLAB, Math Works Inc., Interactive mathematical program.
7. Natke, H.G., Einführung in die Theorie und Praxis der Zeitreihen und Modalanalyse, 3. Aufl., Vieweg Verlag, Braunschweig 1992.
8. Rashid, Y. R., Analysis of Prestressed Concrete Pressure Vessels, *Nuclear Engineering and Design*, Vol. 7 (1968).