MODELING OF PLASTIC MATRIX-FIBER INTERACTION IN FIBER REINFORCED CONCRETE

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SUMMARY

The paper presents a new composite material model, suitable for modeling fiber reinforced concrete under static loading. The governing equations of a new uniaxial rheological device are worked out. The model considers three phases in the composite material, an elastic one, a brittle fracture (concrete), and composite fiber yielding. As for the elastic behavior, the elastic properties can be determined from a simple mixing rule according to the concrete and fiber parameters in the composite. As for the irreversible deformations in the composite material, two permanent variables are introduced, one related to matrix cracking, the other to plastic fiber deformation. The fibers are only significantly activated after cracking due to plastic matrix-fiber interaction, represented in the model by the coupling modulus H. The only parameter to be determined in this model is H, it can be calibrated from experimental test results. In this paper, H is determined from some uniaxial direct tensile test results on notched, Reactive Powder Concrete specimens.

Keywords: fiber reinforced concrete, rheological device, Helmholtz free energy, plastic matrix-fiber interaction, Maxwell-symmetry

1. INTRODUCTION

Steel Fiber Reinforced Concrete (SFRC) is increasingly used as building material in civil engineering structures. Over the last 20 years it has been used for airport runways, tunnels, bridge decks, tubes, hydraulic structures, pipes, dams, industrial floors, etc. The main purpose of applying this material is to increase ductility and fracture toughness of plain concrete. Fibers can limit the crack width and crack propagation in the tension zone.

The efficient use of SFRC materials requires an appropriate modeling of the material behavior, both experimentally and numerically. The main difficulty lies in the unmeasurable concrete and fiber stress distribution related to the complex fiber-matrix

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interaction after cracking. This paper focus on a simple macroscopic constitutive model to account for the plastic matrix-fiber couplings.

2. 1-D RHEOLOGIAL MODEL FOR FIBER-REINFORCED CONCRETE

Figure 1. shows a simple 1-D rheological device for a fiber reinforced composite. The model is composed of two elastic springs (stiffness C_m and C_f, friction strengths k_m and k_f), which describe the elastic matrix (concrete) and fiber behavior, respectively. Fibers are only significantly activated after matrix cracking (k_m) due to plastic matrix-fiber interaction, represented in the model by the spring of rigidity *H*, which links the two components once irreversible deformation occur. The irreversible deformation in the composite material is taken into account by two introduced permanent strain variables, one related to matrix cracking (ε_m^p = plastic matrix strain), the other to plastic fiber strain (ε_f^p). Finally, Σ and ε are the total applied stress and strain (change in length), respectively.

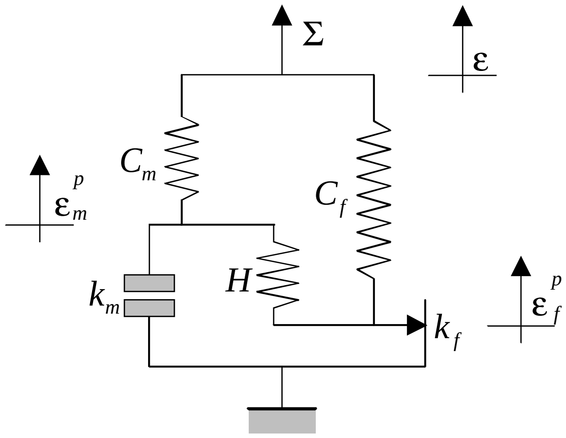

Fig.1 1-D rheological device for fiber-reinforced concrete

Consider now an applied stress, Σ, on the system, which results in composite constituent stresses in the matrix, σ_m, and in the fiber, σ_f, respectively. From elementary equilibrium consideration the force flow in the rheological device is governed by:

$$\Sigma = \sigma_m + \sigma_f = C_m\left(\varepsilon - \varepsilon_m^p\right) + C_f\left(\varepsilon - \varepsilon_f^p\right). \tag{1}$$

$$\sigma_m = C_m\left(\varepsilon - \varepsilon_m^p\right) - H\left(\varepsilon_m^p - \varepsilon_f^p\right), \tag{2}$$

$$\sigma_f = C_f\left(\varepsilon - \varepsilon_f^p\right) + H\left(\varepsilon_m^p - \varepsilon_f^p\right), \tag{3}$$

Eq.(1), (2) and (3) can be expressed in an incremental form. In matrix notation we have:

$$\begin{bmatrix} d\Sigma \\ d\sigma_m \\ d\sigma_f \end{bmatrix} = \begin{bmatrix} C_m + C_f & -C_m & -C_f \\ C_m & -(C_m + H) & H \\ C_f & H & -(C_f + H) \end{bmatrix} \begin{bmatrix} d\varepsilon \\ d\varepsilon_m^p \\ d\varepsilon_f^p \end{bmatrix}. \tag{4}$$

Furthermore, the stresses are constrained by the loading functions:

$$f = max\{f_m, f_f\}; \qquad f_m(\sigma_m) = \sigma_m - f_t \le 0; \qquad f_f(\sigma_f) = \sigma_f - f_y \le 0. \tag{5}$$

Eq.(5) define two yield functions for the composite. Eq.(2), (3) and (5) yield the condition of onset of irreversible matrix or fiber deformation:

$$f = max\{f_m, f_f\} = \sigma_m - f_t \le 0 \qquad if: \qquad f_t / f_y < C_m / C_f, \tag{6}$$
$$f = max\{f_m, f_f\} = \sigma_f - f_y \le 0 \qquad if: \qquad f_t / f_y > C_m / C_f. \tag{7}$$

Finally, to complete the modeling we need to add the evolution laws for both the elastic brittle matrix behavior and for the elastic perfectly plastic fiber behavior (Fig.2):

$$\varepsilon_m^p \ge 0, \qquad \sigma_m = 0, \qquad d\sigma_m = 0; \tag{8}$$
$$\varepsilon_f^p \ge 0, \qquad \sigma_f = f_y, \qquad d\sigma_f = 0. \tag{9}$$

Consider now a fiber reinforced concrete specimen under tension loading. Beyond matrix cracking or fiber yielding, the material behaves elastically, i.e.:

$$f = max\{f_m, f_f\} < 0 \qquad \Leftrightarrow \qquad \Sigma = \sigma_m + \sigma_f = (C_m + C_f)\varepsilon. \tag{10}$$

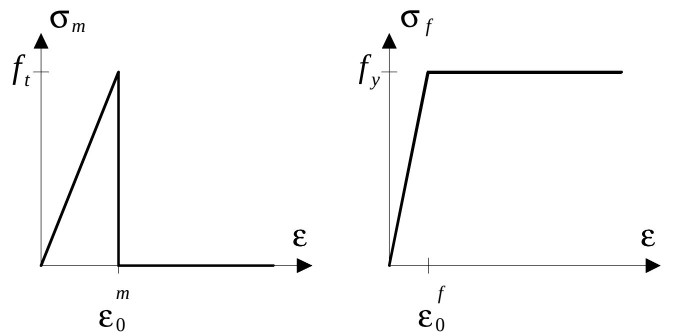

Fig.2: Elastic brittle matrix and elastic perfectly plastic fiber behavior

Generally, in the case of fiber reinforced concrete, matrix cracking occurs prior to fiber yielding. The concrete cracking occurs for $\varepsilon_0 = f_t / C_m$. The following stress state in the composite material defines the matrix cracking:

$$^{-}\varepsilon_m^p = 0 ; \tag{11}$$

$$^{-}\Sigma^0 = \left(C_m + C_f\right)\varepsilon_0 , \quad ^{-}\sigma_m{}^0 = C_m\varepsilon_0 , \quad ^{-}\sigma_f{}^0 = C_m\varepsilon_0 . \tag{12}$$

$$^{+}\varepsilon_m^p = \frac{C_m}{C_m + H}\varepsilon_0 ; \tag{13}$$

$$^{+}\Sigma^0 = \left(C_m + C_f - \frac{C_m{}^2}{C_m + H}\right)\varepsilon_0 , \quad ^{+}\sigma_m{}^0 = 0, \quad ^{+}\sigma_f{}^0 = \left(C_f + \frac{C_f H}{C_f + H}\right)\varepsilon_0 , \tag{14}$$

where the "-" and "+" subscript design the plastic deformations and stresses before and after the matrix cracking, respectively. Due to the plastic matrix-fiber couplings, the matrix cracking leads to a fiber activation. According to the brittle matrix behavior ($\sigma_m = d\sigma_m = 0$), and the equilibrium consideration ($\Sigma = \sigma_m + \sigma_f$) in Eq.(3), the activated fiber stress coincides with the total applied stress: $\Sigma = \sigma_f$. From the onset of cracking, Eq.(6), and the incremental form of the governing equations, (4), lead to the permanent strain increments as:

$$d\varepsilon_m^p = \frac{C_m}{C_m + H} d\varepsilon, \qquad d\varepsilon_f^p = 0 . \tag{15}$$

The stress increment of the composite (respectively of the composite fiber) or the tangential stress-strain relation read:

$$d\Sigma = d\sigma_f = C_{cp} = \left(C_m + C_f - \frac{C_m{}^2}{C_m + H}\right)d\varepsilon = \left(C_f + \frac{C_m H}{C_m + H}\right)d\varepsilon . \tag{16}$$

Finally, the fiber yielding starts for:

$$^{-}\Sigma^1 = {}^{+}\Sigma^1 = {}^{-}\sigma_f{}^1 = {}^{+}\sigma_f{}^1 = f_y , \tag{17}$$

while the plastic strain increment according to the perfectly plastic fiber and brittle matrix behavior reads:

$$d\varepsilon_m^p = d\varepsilon_f^p = d\varepsilon . \tag{18}$$

The determined stress-strain curves for the composite and its constituents are shown in Fig.3. With a simple parameter study of H, it is possible to express two limit cases of the tangential modulus of the composite. For $H = 0$, there is no matrix fiber interaction, while $H \to \infty$ corresponds to a perfect matrix-fiber bond. In these cases, the tangential modulus of the composite materials reduces to:

$$\lim_{H\to 0} C_{cp} = C_f , \qquad \lim_{H\to\infty} C_{cp} = C_f + C_m , \tag{19}$$

and the stress-strain relations:

$$\lim_{H\to 0} d\Sigma = d\sigma_f = C_f\, d\varepsilon , \qquad \lim_{H\to\infty} d\Sigma = d\sigma_f = \left(C_f + C_m\right) d\varepsilon . \tag{20}$$

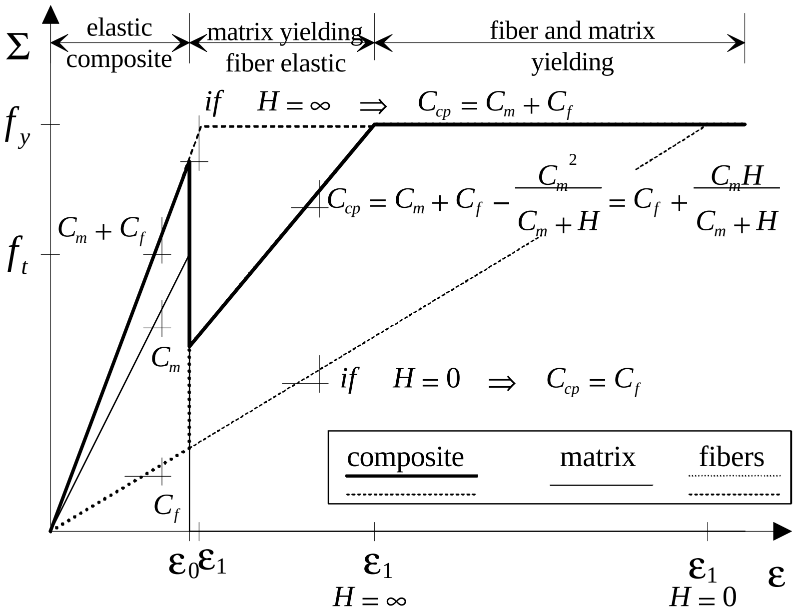

Fig.3 Stress-strain curves for fiber reinforced composite (matrix cracking prior to fiber yielding)

Consider now the special case of fiber yielding prior to matrix cracking. In this case, fiber yielding occurs for $\varepsilon_0 = f_y / C_f$, and the following composite material stress state can be determined at fiber yielding:

$${}^{-}\varepsilon_f^p = {}^{+}\varepsilon_f^p = {}^{-}\varepsilon_m^p = {}^{+}\varepsilon_m^p = 0 , \tag{21}$$

$${}^{-}\Sigma^0 = {}^{+}\Sigma^0 = \left(C_m + C_f\right)\varepsilon_0 \qquad {}^{-}\sigma_m{}^0 = {}^{+}\sigma_m{}^0 = C_m\varepsilon_0 \qquad {}^{-}\sigma_f{}^0 = {}^{+}\sigma_f{}^0 = C_f\varepsilon_0 = f_y , \tag{22}$$

while the permanent strain increment and the tangential stress-strain relation after fiber yielding, according to Eq.(4) and (7) read:

$$d\varepsilon_f^p = \frac{C_f}{C_f + H} d\varepsilon , \qquad d\varepsilon_m^p = 0 , \tag{23}$$

$$d\Sigma = d\sigma_m = \left(C_m + C_f - \frac{C_f{}^2}{C_f + H} \right) d\varepsilon = \left(C_m + \frac{C_f H}{C_f + H} \right) d\varepsilon . \tag{24}$$

Finally, the matrix cracking occurs for:

$$ {}^{-}\sigma_m{}^1 = f_t . \tag{25}$$

The plastic strain increments according to the perfectly plastic fiber and brittle matrix behavior after the matrix cracking is still given by Eq.(18). The following composite stresses at the onset of matrix cracking are hence obtained:

$$ {}^{-}\Sigma^1 = f_t + f_y , \qquad {}^{-}\sigma_m{}^1 = f_t , \qquad {}^{-}\sigma_f{}^1 = f_y \tag{26}$$

$$ {}^{+}\Sigma^1 = f_y , \qquad {}^{+}\sigma_m{}^1 = 0, \qquad {}^{+}\sigma_f{}^1 = f_y . \tag{27}$$

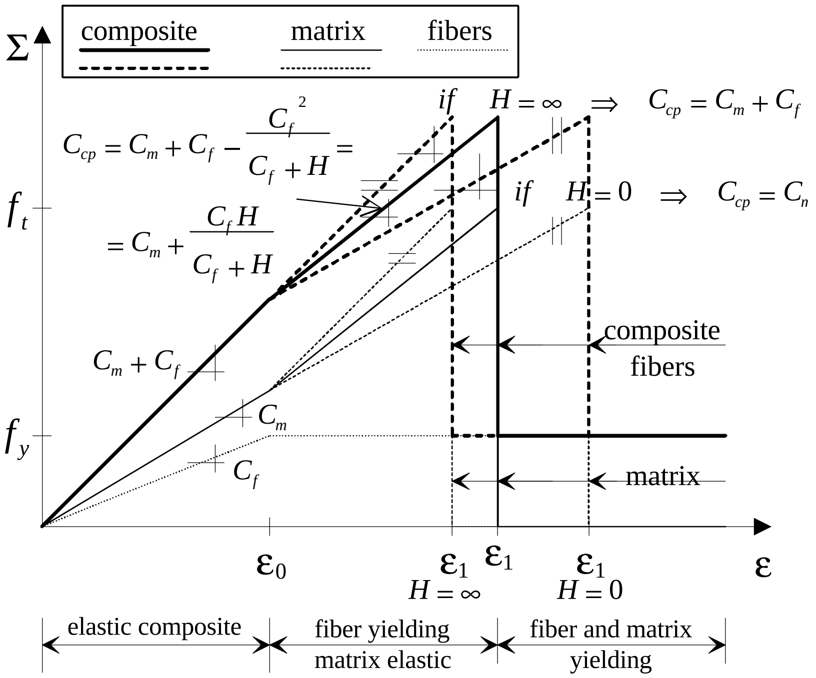

Fig.4 Stress-strain curves for fiber reinforced composite (fiber yielding prior to matrix cracking)

The computed stress-strain curves are shown in Fig.4, where the effect of the coupling modulus H on the composite tangential stress-strain relation can be seen. Two limit values are available for the tangential modulus C_{cp} in this case:

$$ \lim_{H \to 0} C_{cp} = C_m , \qquad \lim_{H \to \infty} C_{cp} = C_f + C_m , \tag{28}$$

and the composite and composite matrix stress increments become now:

$$ \lim_{H \to 0} d\Sigma = d\sigma_m = C_m d\varepsilon , \qquad \lim_{H \to \infty} d\Sigma = d\sigma_m = \left(C_f + C_m\right) d\varepsilon . \tag{29}$$

3. IDENTIFICATION OF COMPOSITE MATERIAL PARAMETERS

The introduced material parameters are composite material parameters which cannot, without any further investigation and hypothesis, be directly associated with material parameters of the materials constituting the composite. In this paper for sake of simplicity, we consider the simplest identification process, using a parallel mixture rule for the elastic properties, i.e.:

$$C_m = (1-\eta_f)E_m, \qquad C_f = \eta_f E_f, \tag{30}$$

where η_f =fiber content, E_m and E_f =elastic modulus of concrete and of the fiber material, respectively. The composite strengths may be determined in a similar way. In other words, the only undetermined parameter of the model is *H*, it can be calibrated from direct tensile tests.

In the present work, *H* is determined from direct tensile test results on notched Reactive Powder Concrete specimens. The material parameters for the applied two RPC mixtures are given in Tab.1.

	Young's modulus	*Compressive strength*	*Flexural strength*	*Fracture energy*	*Fiber content*
RPC200	50…60GPa	170…230MPa	30…60MPa	20…40KJm^{-2}	1.5 V%
RPC800	65…75GPa	490…680MPa	45…140MPa	1.2…20KJm^{-2}	3 V%

Tab.1 Material parameters for RPC200 and RPC800, Richard, P., Cheyrezy, M.(1995)

Fig.5 shows a good agreement between test results and model assumption. Note, the ultimate macroscopic strain of composite was calculated according to the energy absorption of composite (area under the curve). Both for RPC200 and RPC800, the moduli of plastic couplings was *H*=1600MPa. This result suggest a strong dependence of *H* on the microstructure of concrete (aggregates, fibers, etc.). The determination of this effect needs further experimental and theoretical investigation.

4. CONCLUSIONS

A new material model for fiber reinforced concrete is presented in this paper which takes into account plastic matrix-fiber couplings. As the results suggest, the model can capture the essential features of the fiber reinforced concrete and of its constituents under tension loading. The main advantage of this model is that the only undetermined material parameter, H, can be calibrated from uniaxial tension tests. The determination of the dependence of H on the constituents material parameters and on the microstructure needs further intensive experimental and theoretical investigation.

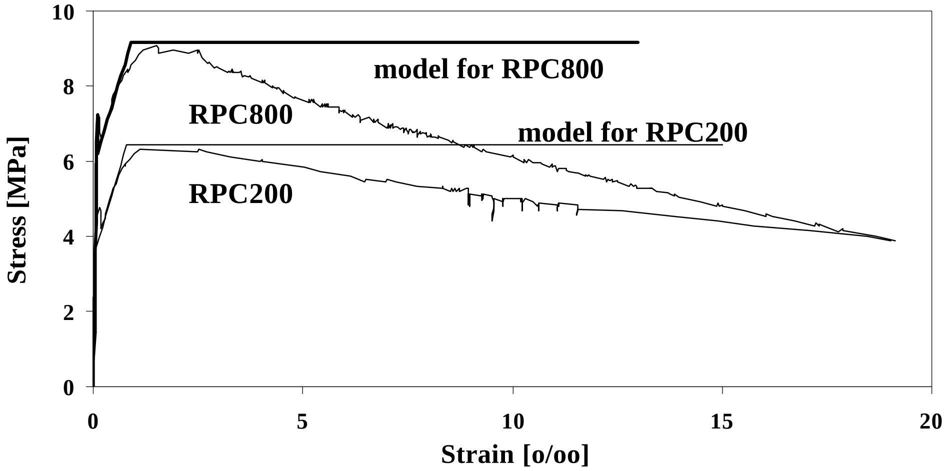

Fig.5 Tensile stress-strain curves for RPC200 and RPC800

5. FUTURE WORK

In the simplified 1-D case one array of fibers was only considered. The model can be extended to the 3-D case with multidirectional arrays of fiber. To this end, an energetic approach can be used, which allows for a straightforward extension of the 1-D model to the 3-D case. Furthermore, the 1-D rheological device can easily be extended with softening and hardening behavior of matrix and fibers as well. This modification leads to a more complex rheological model of fiber reinforced concrete under static loading. Finally, the developed material model allows us to work out a simple bending model for SFRC elements based on the equilibrium of forces in a cracked section.

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