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DETERMINATION OF STRESS INTENSITY FACTORS ON ROCK SPECIMENS

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SUMMARY

The aim of this research is to determine stress intensity factors (K_I) of carbonate sedimentary rock-beams under three point bending. To determine this significant **toughness** property experimental tests have been carried out on half-cylindrical specimens. These specimens can be prepared more easily from cylindrical boring core and have less loss than the widely used rectangular type. This paper presents the experimental and numerical investigations for both rectangular and half-cylindrical specimens, and suggests analytical formula for calculation of stress intensity factor based on the new geometry.

Keywords: stress intensity factor, finite element method, rock mechanics

1. INTRODUCTION

A great deal of experimental researches in the field of rock mechanics are strongly connected to fracture and failure of different type of rocks and concentrate on determination of material properties, amongst which **stress intensity factor** (*K*) is the most useful and significant one for qualification of rocks on the basis of their toughness.

Selection of the specimen type used for the experiments is an important question. Available laboratory equipment and measurement technics determine geometric formation of specimens. At rock mechanical investigations the most often used sample is **the core material** prepared from trial boring. Deep borings provide high-level information on the rock environment of engineering structures. Application of boring core material serves as an economic source to gain regular cylindrical specimens, which are produced by cutting it into slices.

Usage of cylindrical specimens made of boring core material has been previously suggested by other authors. **Ouchterlony** (1989) tested full cylindrical specimens with notches of special shape to determine *K* values. "Modified disc specimens" (notched disc) can also be used for fracture mechanical investigations. **Czoboly et al.** (1986) put forward a proposal for future adaptation of these specimens. For the current research specimens produced with the easiest procedure have been used for experiments: **the half-cylindrical specimen** can be produced by cutting the boring core into two parts

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along its longitudinal axis. Certainly the shape of this specimen raises a large number of technical questions. This paper gives answers to these with demonstrating the results of the theoretical and experimental research.

The purpose of our research was to develop a new type specimen, that can be produced easily and available in large quantities, and make it suitable for determination of fracture mechanical properties, thus these specimens can be fitted into the system of the rock mechanical laboratory experiments.

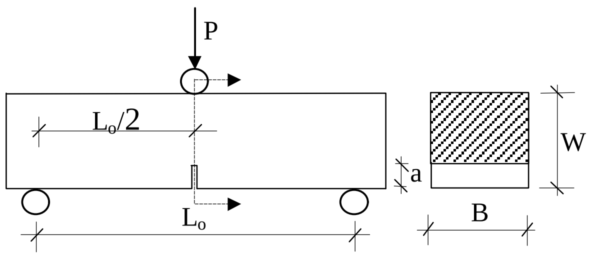
2. LABORATORY EXPERIMENTS

Three point bending tests were done on specimens with different cross-sections in the laboratory of Dept. of Engineering Geology at Technical University of Budapest. The specimens were made of sandstone, dense and coarse limestone. At each stone first **rectangular**, then **half-cylindrical** specimens were broken. For experimental series those rocks had been chosen, that satisfied the prescribed requirements of homogeneous and isotropic conditions. The examined rocks were:

- **dense limestone** (**I.**) (Budapest, Pesthidegkút, Hungary) with carbonate tissue and partially containing calcite- and clay-veins,
- **coarse limestone** (Vraca, Bulgaria), fine-grained, carbonatic rock with beet potashed bonding,
- **sandstone** (Maulbronn, Germany), auburn, fine-grained rock with glued tissue and silicate bonding,
- **dense limestone** (**II.**) (Vác, Hungary) fine-grained, compact rock with micro-crystal tissue and containing calcite-knots.

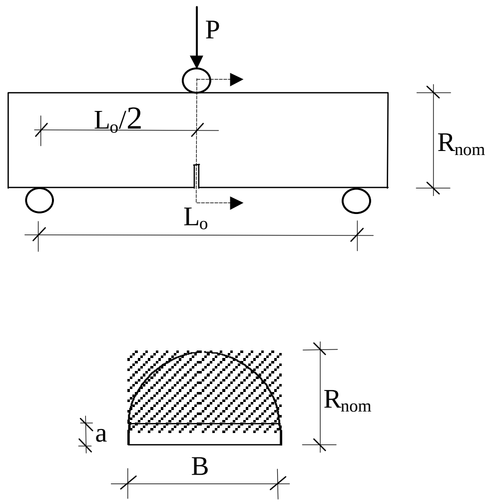
The shape of the artificial notch causing crack and "collecting" tensile stress concentration was the same for all types of rock material and geometric shapes. The measurements were done on laboratory air-dried rock physical specimens. In every case the experimental apparatus was a universal compress machine type SZF1.

The geometrical characteristics, data and the type of loading are shown in Fig.1 at rectangular and Fig.2 at half-cylindrical specimens. It is important to mention that rectangular specimens were made from the same cylindrical cores as we used for the production of the half-cylindrical samples. The basic strength properties (Young's modulus, Poisson's ratio) were also gained from the laboratory experiments. Tab.1 shows these values for the four rock types.



Sample	L_o [mm]	W [mm]	B [mm]	a [mm]
PI-1	100	33,5	36,7	4,36
PI-2	100	39,6	45,4	6,61
PI-3	100	38,7	39,4	3,63
PI-4	100	39,6	38,9	4,12
PI-5	100	39,8	39,2	4,90
PI-6	100	39,9	39,2	1,54

Fig.1 Geometrical characteristics of the rectangular specimen



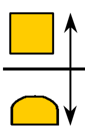
Sample	L_o [mm]	R_{nom} [mm]	B [mm]	a [mm]
CI-1	100	43,3	83,8	2,42
CI-2	100	37,8	83,3	1,48
CI-3	100	38,0	83,2	2,79
CI-4	100	27,3	55,1	1,87
CI-5	100	25,3	54,6	2,06
CI-6	100	26,9	58,9	1,64
CI-7	100	29,9	59,3	2,24
CII-1	100	37,5	63,4	3,80
CII-2	100	30,8	63,0	2,53
CII-3	100	31,8	63,3	3,58
CII-4	100	32,0	63,5	3,06
CII-5	100	30,0	63,3	3,78
CII-6	100	30,9	62,6	2,93
CII-7	100	34,3	71,5	4,69
CII-8	100	21,3	53,9	4,96
CII-9	100	26,1	55,1	3,99
CII-10	100	21,6	54,2	5,63
CII-11	100	26,5	55,1	4,01

Fig.2 Geometrical characteristics of the half-cylindrical specimen

Based on experimental results critical load was determined, that led to bending failure. Loads causing fracture of rectangular and half-cylindrical specimens can be seen in Tab.2.

Sample	Material	Young's modulus (E) [N/mm^2]	Poisson's ratio (ν) [-]
PI-1, PI-2; CI-1 - CI-3	Dense Limestone (I.)	26340	0,20
PI-3, PI-4; CI-4, CI-5	Coarse Limestone	8430	0,25
PI-5, PI-6; CI-6, CI-7	Sandstone	7830	0,25
CII-1 - CII-11	Dense Limestone (II.)	35870	0,24

Tab.1 Strength properties



Sample	Material	Critic. load (P_c) [N]	Sample	Material	Critic. load (P_c) [N]
PI-1	Dense Limest. (I.)	2450	CII-1	Dense Limest. (II.)	2920
PI-2	Dense Limest. (I.)	5000	CII-2	Dense Limest. (II.)	3400
PI-3	Coarse Limest.	410	CII-3	Dense Limest. (II.)	2280
PI-4	Coarse Limest.	380	CII-4	Dense Limest. (II.)	3060
PI-5	Sandstone	1150	CII-5	Dense Limest. (II.)	2630
PI-6	Sandstone	1460	CII-6	Dense Limest. (II.)	3900
CI-1	Dense Limest. (I.)	7650	CII-7	Dense Limest. (II.)	4000
CI-2	Dense Limest. (I.)	7400	CII-8	Dense Limest. (II.)	900
CI-3	Dense Limest. (I.)	6850	CII-9	Dense Limest. (II.)	1720
CI-4	Coarse Limest.	190	CII-10	Dense Limest. (II.)	850
CI-5	Coarse Limest.	140	CII-11	Dense Limest. (II.)	1750
CI-6	Sandstone	760			
CI-7	Sandstone	890			

Tab.2 Critical load values

3. NUMERICAL SOLUTIONS

The regulations based on fracture mechanics' theoretical methods give possibility for the determination of stress intensity factor limit with the help of critical load determined by laboratory measurements. These relations are based on the **Irwin's** theory that modified the **Koloszov-Muszhelisvili-Westergaard's** disc-solving executed with complex stress functions. These discs had an artificial sharp crack and supposed infinite size. Originally Irwin restricted the solution valid for the whole disc to the immediate surroundings of the crack tip. The stress intensity factor introduced in the course of this reduction is used for mathematical description of **singularity** and physical description of **stress concentration** (see the general works listed at References, e.g.: Broek, Hahn or Anderson), and its limit can be considered as a **material constant** at a given geometrical crack face.

The situation at real structures is much more complicated than as mentioned above: boundary conditions provided by finite size and complicated geometrical form is basically different from the assumptions of the original solution. In practical cases this difficulty is absolved in such a way that size- and shape-dependent correction factors are initiated to modify the results derived from theoretical assumptions. This leads to the consequence that new geometrical **correction** factors have to be determined for each new case (see „Stress Intensity Factors Handbook" published in Japan and the U.S.A.). These effects are the reasons for the necessity of the numerical examinations connected to the experiments performed by us, as the present norm gives the geometrical correction factors' value only for the three point bending test of rectangular specimen. Therefore these factors have to be determined separately at half-cylindrical specimens. At the present examinations we used fracture mechanical applications provided by **three dimensional** (3D) **finite element analysis**.

First the critical stress intensity factors (K_{Ic}) were calculated at rectangular specimens by means of a suitable **analytic formula** chosen from the current **standard**. Tab.3 shows the applied formula and the results.

Sample	Material	K_{Ic} [N/mm$^{3/2}$]
PI-1	Dense Limestone (I.)	58,12
PI-2	Dense Limestone (I.)	83,86
PI-3	Coarse Limestone	6,31
PI-4	Coarse Limestone	5,99
PI-5	Sandstone	19,23
PI-6	Sandstone	14,56

$$K_{Ic} = \frac{3 \cdot P_c \cdot L_o}{2 \cdot B \cdot W^2} \cdot \sqrt{\pi \cdot a} \cdot Y(\alpha) , \qquad \text{where} \qquad \alpha = \frac{a}{W} , \qquad (1)$$

$$Y(\alpha) = \frac{1{,}99 - \alpha \cdot (1-\alpha) \cdot \left(2{,}15 - 3{,}93 \cdot \alpha + 2{,}70 \cdot \alpha^2\right)}{(1 + 2 \cdot \alpha) \cdot (1-\alpha)^{3/2}}$$

Tab.3 Stress intensity factors of rectangular specimens

Next we examined the results of 3D finite element analysis. Fig.3 and Fig.4 show the applied finite element meshes at the rectangular and the half-cylindrical specimens.

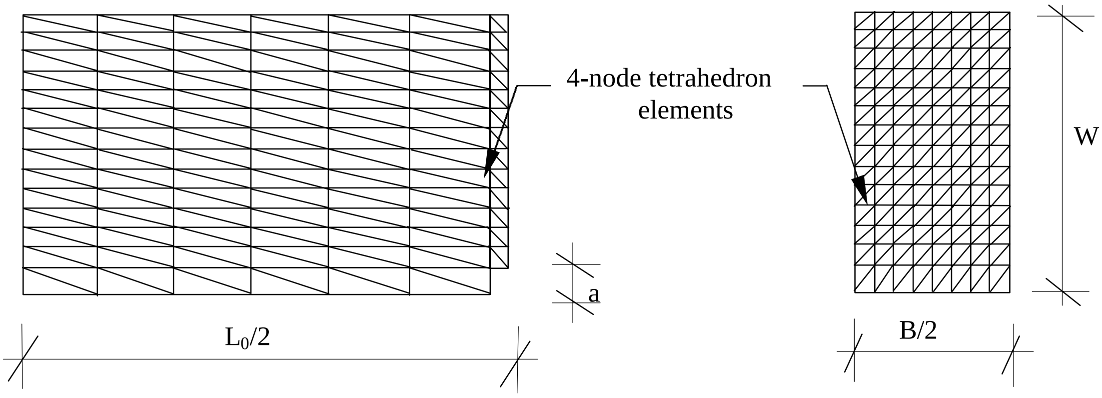


Fig.3 3D finite element mesh of rectangular specimens

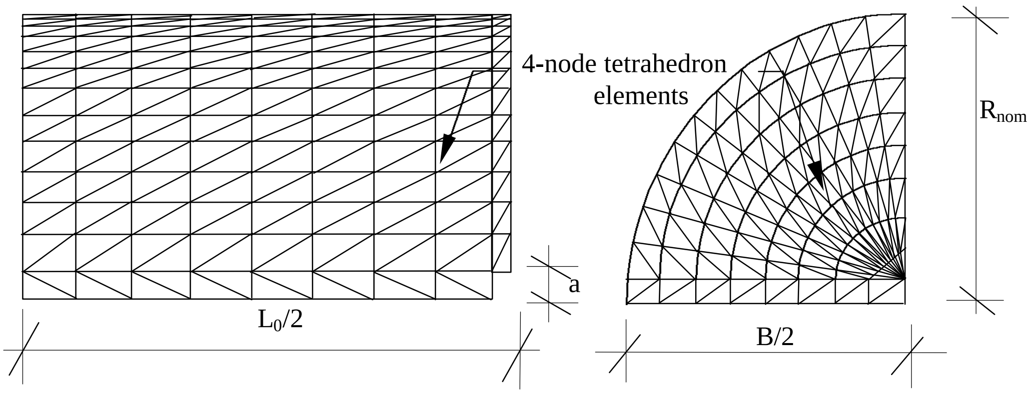


Fig.4 3D finite element mesh of half-cylindrical specimens

Beside the finite element mesh a typical stress distribution of the vertical segment under the load can be seen for half-cylindrical specimens in Fig.5.

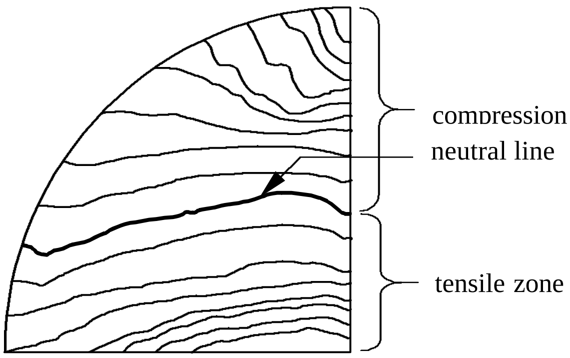


Fig.5 Contour line picture of stress distribution under the load

During fracture mechanical solutions the finite element investigation give estimation for the stress intensity factor with the application of displacement field (see Koloszov-Muszhelisvili's work) at the surroundings of the crack tip/artificial notch. The connection of K_{Ic} values with displacements can be described with the following simple equation:

$$K_{Ic} = \frac{E}{1-\nu^2} \cdot \sqrt{\frac{\pi}{2 \cdot r_i}} \cdot \frac{v_i}{2} \; , \qquad \text{where}$$

v_i is the displacement component in the direction of notch, r_i is a polar co-ordinate which origin is at the crack tip, E is the Young's modulus, n is the Poisson's ratio. Naturally this relation is valid only far from the crack tip, so at the tip, at the singularity place we got the K_{Ic} value from the extrapolation of the other K_{Ici} values. Fig.6 shows a sample for this extrapolation e.g. based on data of CI-1 specimen.

i	r_i [mm]	K_{Ici} [N/mm$^{3/2}$]
221	5,11	139,19
208	10,22	100,58
195	15,33	84,31
182	20,44	76,44
169	25,55	74,49
156	30,66	80,11
143	35,77	99,76
130	40,88	186,36

$\Rightarrow$ K_{Ic}= 78,84 N/mm$^{3/2}$

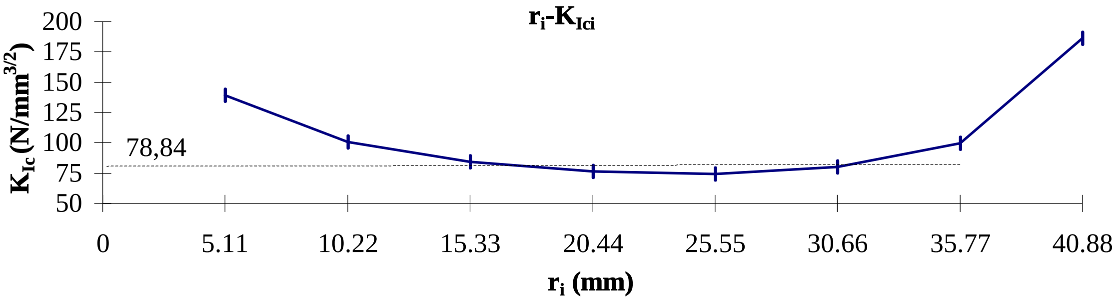


Fig.6 Sample for the extrapolation of stress intensity factor (CI-1 specimen)

The K_{Ic} values got from the results of 3D finite element solutions can be seen in Tab.4.

Sample	Material	K_{Ic} [N/mm$^{3/2}$]	Sample	Material	K_{Ic} [N/mm$^{3/2}$]
PI-1		65,43	CII-1		52,47
PI-2		80,09	CII-2		96,86
CI-1	Dense Limestone (I.)	78,84	CII-3		61,84
CI-2		101,25	CII-4		80,01
CI-3		95,75	CII-5		84,10
PI-3		7,53	CII-6	Dense Limestone (II.)	112,36
PI-4	Coarse Limestone	6,77	CII-7		84,57
CI-4		8,18	CII-8		94,96
CI-5		7,56	CII-9		91,58
PI-5		20,33	CII-10		88,64
PI-6	Sandstone	24,38	CII-11		88,85
CI-6		31,98			
CI-7		28,60			

Tab.4 Extrapolated values

4. THE NEW ANALYTIC FORMULA

Our next task was to determine a new analytic formula for the half-cylindrical specimens to give the same results for the stress intensity factor like as we got at the rectangular ones. We suggest the following new relation:

$$K_{Ic} = \frac{P_c \cdot L_o}{R_{nom}^3} \cdot \sqrt{4 \cdot \pi \cdot a} \cdot Y(\alpha) \ , \qquad \text{where} \qquad \alpha = \frac{a}{R_{nom}} \ , \qquad (2)$$

$$Y(\alpha) = 1{,}52 - 2{,}20 \cdot \alpha + 7{,}71 \cdot \alpha^2 - 13{,}55 \cdot \alpha^3 + 14{,}25 \cdot \alpha^4 \ ,$$

R_{nom} is the height of specimen, L_o is the distance between the supports, a is the crack length and P_c is the critical load. The coefficients of the *Y* polynome were determined with the help of less square method. The K_{Ic} soluted from this new formula were compared with the results of the other methods (see Tab.5).

Sample	New K_{Ic} ($N/mm^{3/2}$)	K_{Ic} from num. solution ($N/mm^{3/2}$)	Difference (%)
CI-1	73,73	78,84	6,48
CI-2	85,37	101,25	15,68
CI-3	103,12	95,75	7,15
CI-4	6,34	8,18	22,49
CI-5	6,09	7,56	19,44
CI-6	25,02	31,98	21,76
CI-7	24,61	28,60	13,95
CII-1	52,18	52,47	0,55
CII-2	90,84	96,86	6,22
CII-3	64,34	61,84	3,89
CII-4	79,30	80,01	0,89
CII-5	90,07	84,10	6,63
CII-6	109,91	112,36	2,18
CII-7	101,49	84,57	16,67
CII-8	95,33	94,96	0,39
CII-9	90,64	91,58	1,03
CII-10	91,96	88,64	3,61
CII-11	88,39	88,85	0,52
		Mean difference:	**8,31**

Tab.5 Comparison of the numerical and analytical values

The low error-percentage difference (shown in Tab.5) refers to the serviceability of the *(2)* formula.

In the last step, besides checking the correction of the new formula, the effect of crack length to KIc values (determined from *Eq(2)*) was examined at half-cylindrical dense limestone specimens (signed CII-i). The results of this investigation can be seen in Fig.7. The value of K_{Ic} stabilizes with the increase of the relative crack length at 0,15.

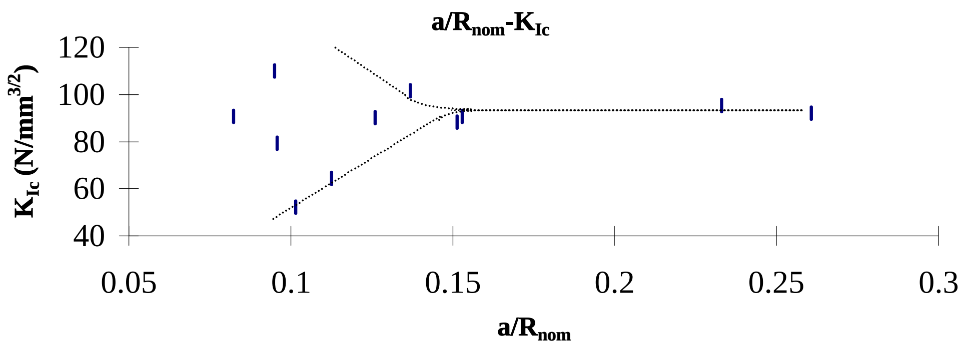


Fig.7 Effect of the crack length to the stress intensity factor

5. CONCLUSIONS

As a result of this research a new analytic formula (*Eq(2)*) has been created for stress intensity factor at half-cylindrical specimen, which can be produced more easily at rocks than the widely used and standardized rectangular type. First, critical failure loads were measured on both types of specimens made of sandstone, dense and coarse limestone in three point bending. Based on these values stress intensity factors of rectangular specimens were calculated from available standardized analytical formula (*Eq(1)*), then finite element analysis was performed in order to evaluate characteristic displacement values and stress intensity factors numerically. As a result of data analysis a new analytic formula (*Eq(2)*) was created for half-cylindrical specimens made of carbonate sedimentary rocks. Testing of the new formula provided low error-percentage difference (~8%), that proved its reliability.

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